Competencies of Math 15

Student Workbook

CALGARY CATHOLIC SCHOOL DISTRICT

Summer School 2017
Competencies of Math 15
Student Workbook:

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JULY 2017

Competencies of Math 15

Sunday  | Monday  | Tuesday  | Wednesday | Thursday | Friday  | Saturday
---      |---       |---       |---        |---       |---      |---
25       | 26       | 27       | 28        | 29       | 30      | 1

2   | 3   | 4   | 5   | 6   | 7   | 8
Intro to Decimals | Number Systems | Exponents | Factors & Products | Review | Unit Test |
Decimal Operations | Integers | Order of Operations | GCF/LCM |           |          |

9   | 10  | 11  | 12  | 13  | 14  | 15
Frac,Deci,% | Frac,Deci,% | Fraction Operations | Review | Coordinate System & Ordered Pairs | |
Equivalent Fractions |           |               | Unit Test |           |          |

16  | 17  | 18  | 19  | 20  | 21  | 22
T of Values Equations | Slope (+/-/v/h) | Solving Linear Equations | Review | Polynomial definitions | Like Terms |

23  | 24  | 25  | 26  | 27  | 28  | 29
Multiplying Polynomials | Dividing Polynomials | Review | Course Review | Final Exam |          |

30  | 31  | 32  | 33  | 34  | 35  | 36

Course Information:

Classes run: Tuesday July 4th to Friday July 28th
8:25am to 12:58pm
Competency Outcomes:

1. Students will communicate mathematical ideas in a variety of ways and contexts, and begin to view mathematics as useful and relevant by making connections to other disciplines.
   1.1 Communicate effectively using the language of mathematics.
   1.2 Apply language, knowledge, and strategies to build common understandings across disciplines
2. Students will gain knowledge, understanding, and skills through study and interaction with others.
   2.1 Implement and refine strategies to maximize learning in a variety of authentic learning situations.
   2.2 Apply knowledge of patterns, number, shape, space, statistics and probability to help me observe, investigate and interact with the world.
   2.3 Apply efficient and mental calculation strategies intuitively when solving complex problems
3. Students will identify and solve complex problems.
   3.1 Establish clear criteria to solve problems.
   3.2 Develop and apply problem solving strategies to generate possible solutions using a variety of techniques, strategies, and processes.
   3.3 Develop the best possible solution by evaluating the validity of alternate solutions.
4. Students will be able to think critically and use mathematical reasoning to make sense of mathematics.
   4.1 Express generalizations about numbers, quantities, and relations and functions when analyzing data.
   4.2 Analyze patterns effectively to identify rules and trends and make predictions.
   4.3 Evaluate reasoning and strategies used in the problem-solving process.
5. Students will select and apply multiple literacies to solve problems and to enhance learning.
   5.1 Demonstrate effective use of technology as a problem-solving tool.
   5.2 Simplify complex problems through the use of technology.
   5.3 Integrate multiple literacies in the problem-solving process.
   5.4 Demonstrate the use of concrete materials, technology, and visual representations to solve problems.

Competency Units of Study:

1. Number Sense
   - Integers, Powers & Exponents, Fractions & Decimals
   - Order of Operations
   - Factors & Products (GCF & LCM)

2. Coordinates & Equations
   - Cartesian Plane, Coordinate Points
   - Equations of lines & slope
   - Solving linear equations & Isolating variables

3. Polynomials
   - Adding & Subtracting polynomials
   - Multiplying & Dividing polynomials
Course Evaluation:

Number Sense 40%
Coordinates & Equations 20%
Polynomials 20%
Final Exam 20%

Evaluation within each Unit will consist of a variety of assessment methods; including daily Outcome Based Check-Points, Unit Tests, Quizzes, Projects and Hands-on Activities. If a student misses an assessment they will write the assessment upon returning to class or at a time pre-arranged with the teacher.

Re-assessment opportunities will be provided to students throughout the course. Specifics will be discussed with students on an individual basis to address specific student needs. If you have any questions, please feel free to call/email your teacher.

Materials:

Provided: Competencies of Math 15 Workbook

You must provide: One of the two calculators listed below:
Scientific Calculator; TI30X-IIS (minimum requirement)
TI-83/84 Plus Graphing Calculator (this calculator is required for Math 10C)

NOTE:
This course does not count towards your high school math credits (10 required for graduation); however does count towards the 100 credits required for high school graduation.

* Students must pass Grade 9 Math or Competencies of Math 15 to enrol in Math 10C.

Teacher Name
Teacher Email
School Phone Number
Unit 1A:
Number Sense
Unit 1: Number Sense

Lesson 1

Number Systems and Ordering

Concepts
- Introduction to the Real Number System
- Identify Natural, Whole, Integer, Rational and Irrational numbers into their correct classification
- Represent numbers in the real number system on number lines, charts and tables

Self-Check
☐ I understand how to classify Natural Numbers and where they came from
☐ I can explain Whole Numbers about classify numbers
☐ I can identify Integers and explain the properties
☐ I know the difference between Rational Numbers and Irrational Numbers

Definitions:

The Real Number System:

Natural Numbers (N)
The first group of numbers in the Real Number Systems are natural numbers. These are the counting numbers starting at 1. \{1, 2, 3, 4, …\}.
Natural Numbers are represented by \(N\).

Whole Numbers (W)
The second group of numbers is the set of Whole Numbers. These are the counting numbers starting at 0. \{0, 1, 2, 3, 4 …\}.
The set of Whole Numbers is represented by \(W\).

Integers (I)
The third group of numbers are Integers. Integers are the set of counting numbers, positive and negative. The set of Integers are represented by \(I\). \{-3, -2, -1, 0, 1, 2, 3…\}

Rational Numbers (Q)
The fourth group of numbers are Rational Numbers. Rational Numbers can be positive or negative, and include numbers that can be represented by a fraction.
The set of numbers \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b \neq 0\).
This includes decimals that either terminate, or repeat. \{\(\frac{2}{3}\), 0, 1.2, -8\}.

Irrational Numbers (\(\mathbb{Q}\))
Irrational Numbers are separate from the other four systems, but still a member of real numbers. These are numbers such as \(\pi\). These numbers have decimals that do not repeat and do not terminate.
**The Real Number System (R)**

VIDEO: [https://www.youtube.com/watch?v=m94WTZP14SA](https://www.youtube.com/watch?v=m94WTZP14SA)

---

**Class Examples:**
Classify the following numbers into their correct number system(s):

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>π</td>
<td>√6</td>
</tr>
</tbody>
</table>

---

**Try these on your own:**
Classify the following numbers into their correct number system(s):

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>−3</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1.423...</td>
<td>√3</td>
</tr>
</tbody>
</table>

---
Number Systems and Ordering

1. Name the sets of numbers to which each number belongs
   a) \( \sqrt{12} \) a) 0
   b) \(-7\) b) 2.75
   c) 1 c) \( \pi \)

2. Identify if the following are rational or irrational
   d) \( \frac{2}{5} \) d) \(-15\)
   e) \( \sqrt{35} \) e) \( \sqrt{49} \)
   f) 5 f) \( \frac{1}{10} \)

3. Classify the following the values into the appropriate number system in the real number system

The Real Number System

\begin{itemize}
  \item a. -13
  \item b. 3.25
  \item c. 15
  \item d. -1.111...
  \item e. 0
  \item f. 100
  \item g. \( \pi \)
  \item h. 30
  \item i. \( \sqrt{16} \)
  \item j. \(-\sqrt{4} \)
  \item k. \( \sqrt{-10} \)
\end{itemize}
Unit 1: Number Sense
Lesson 2

Introduction to Integers

Concepts
- Identify everyday scenarios that would result in positive and negative integer values.
- Graph Integers on a number line.
- Demonstrate and understand of addition and subtraction of integers, concretely, pictorially and symbolically.
- Demonstrate and understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Self-Check
- I can add integers
- I can subtract integers
- I can multiply integers
- I can divide integers
- I can solve problems using integers

**Definition: Integers**

The set of **integers** is symbolized by the capital letter \( I \) and us composed of all the **positive numbers** and all the **negative numbers** and also the number **zero** which is considered neutral (Neither positive, nor negative).

\[
I = \{ \ldots, -4, -3, -2, -1, 0, +1, +2, +3, +4, \ldots \}
\]

We read the numbers as:

\(+8\)  positive eight

\(-5\)  negative five

The number \(0\) (Zero) is neutral, meaning it has no positive or negative sign in front of it.

Integers are used to express many things in our daily lives including temperatures, elevations, golf scores, profits, losses, financial statements, and various other things.

A profit of $700.00 can be written as +700 whereas a loss of $700.00 can be written as –700.

**Class Examples:**
Write an integer to represent each of the following.

| A golfer scores 4 under par. | The elevator rose 7 floors. |

**Try on your own:**
Write a scenario that could represent the integer listed.

| –12 | +22 |
1. Indicate whether each would result in a positive or negative value.
   a) Spending $27
   b) Falling off a ladder
   c) Depositing $450
   d) Climbing a mountain
   e) Going from 5th to 2nd floor
   f) Drop in temperature

2. Write an integer to represent each of the following.
   a) Losing $10
   b) Making a profit of $250
   c) Six more than zero
   d) Finding a $100 bill
   e) Temp. decrease of 3 degrees
   f) 7 less than zero

3. Graph each of the following the number lines provided.
   a) $-6, +5, -4, +2, 0$
   b) $-2, 2, 0, 4, -1$

4. Write the three integers that would appear next in the following sequences.
   a) $+2, +4, +6, \ldots$
   b) $-9, -5, -3, \ldots$
   c) $-11, -6, -1, \ldots$
   d) $14, 9, 4, \ldots$
   e) $-2, -6, -11, -17, \ldots$

5. Write out the set of integers that are described in each.
   a) Integers greater than 0 but less than +5
   b) Integers greater than $-4$ but less than +6
   c) Integers less than $-3$ but greater than $-9$
Operations of Integers

Adding Integers:
When the signs are the same; keep the sign you begin with.

\[(+2) + (+3) = +5\]
Positive + Positive = Positive

\[(-1) + (-5) = -6\]
Negative + Negative = Negative

Add the numbers together and keep the positive sign

Add the numbers together and keep the negative sign

When the signs are opposite subtract the values and keep the sign of the larger value.

\[(-7) + (+3) = -4\]
Negative value is larger, attach the negative sign

\[(+10) + (-1) = +9\]
Positive value is larger, attach the positive sign

Class Examples:
Solve each of the following:

<table>
<thead>
<tr>
<th>(+8) + (+12) =</th>
<th>(−3) + (−6) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+10) + (−4) =</td>
<td>(−15) + (6) =</td>
</tr>
</tbody>
</table>

Try on your own:
Solve each of the following

<table>
<thead>
<tr>
<th>(−9) + (−7) =</th>
<th>(+4) + (−5) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−9) + (+2) =</td>
<td>(−3) + (+14) =</td>
</tr>
</tbody>
</table>
**Subtracting Integers:**

When subtracting integers keep the sign of the first integer, then add the opposite.

Another way to remember this rule is: Keep → Change → Change

\[
(-3) - (+6) \\
(-3) + (-6) \quad \text{Keep (-3) Change - to + Change (-6)} \\
(-3) + (-6) = -9 \quad \text{Use adding rules to evaluate}
\]

**Class Examples:**

Solve each of the following:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+4) − (+3) =</td>
<td>(-6) − (-4) =</td>
</tr>
<tr>
<td>(+6) − (-12) =</td>
<td>(-5) − (+7) =</td>
</tr>
</tbody>
</table>

**Try on your own:**

Solve each of the following

<table>
<thead>
<tr>
<th>Equation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-19) − (-18) =</td>
<td>(-4) − (+2) =</td>
</tr>
<tr>
<td>(+11) − (+14) =</td>
<td>(+40) − (+3) =</td>
</tr>
</tbody>
</table>
**Multiplying Integers**

*Product* is the result of a multiplication.
For example, the product of 3 and 4 is 12.

**Steps:**

1. Find the sign part of the answer.
   
   $$(+)(+) = (+)$$  
   $$(+)(-) = (-)$$  
   $$(-)(-) = (+)$$  
   $$(-)(+) = (-)$$

2. Find the product by multiplying the two numbers together.

3. State the final answer using both the sign and numeric value.

**Example:**

$$(+9)(+3)$$

**Step One:** Find the sign part of the answer.

$$(+)(+) = (+)$$

**Step Two:** Find the number part of the answer.

$$9 \times 3 = 27$$

**Step Three:** State the final answer.

$$(+9)(+3) = +27$$

$$(−7)(+5)$$

**Step One:** Find the sign part of the answer.

$$(-)(+) = (-)$$

**Step Two:** Find the number part of the answer.

$$7 \times 5 = 35$$

**Step Three:** State the final answer.

$$−35$$
**Class Examples:**
Solve each of the following:

<table>
<thead>
<tr>
<th>(+6)×(+3) =</th>
<th>(−8)(−5) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+10)×(−6) =</td>
<td>(−4)(+5) =</td>
</tr>
</tbody>
</table>

**Try on your own:**
Solve each of the following

<table>
<thead>
<tr>
<th>(+9)×(−4) =</th>
<th>(+14)×(+2) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−9)×(−3) =</td>
<td>(−5)×(1) =</td>
</tr>
</tbody>
</table>
**Dividing Integers**

*Quotient* is the result of a division.
For example, the quotient of 36 divided by 9 is 4.

**Steps:**
1. Find the sign part of the answer.
   
   
   
   
   (+) ÷ (+) = (+)  
   
   
   
   
   (−) ÷ (−) = (+)  
   
   
   
   
   (+) ÷ (−) = (−)  
   
   
   
   
   (−) ÷ (+) = (−)

2. Find the quotient by dividing the 2 numbers.

3. State the final answer (sign and numeric value).

**Example:**

\[
\begin{array}{c}
-44 ÷ +2 \\
\hline
-70 \\
-7
\end{array}
\]

**Step One:** Find the sign part of the answer.

\[
\begin{align*}
(−) ÷ (+) &= (−) \\
(−) ÷ (−) &= (+)
\end{align*}
\]

**Step Two:** Find the number part of the answer.

\[
\begin{align*}
44 ÷ 2 &= 22 \\
70 ÷ 7 &= 10
\end{align*}
\]

**Step Three:** State the final answer.

\[
\begin{align*}
44 ÷ 2 &= -22 \\
+10
\end{align*}
\]

**Note:** Zero divided by any integer (+ or -) is always 0. Any integer (+ or -) is always undefined. Division by zero is NOT allowed.

**Class Examples:**

Solve each of the following:

\[
\begin{array}{c|c}
(+15) ÷ (+5) = & -18 \\
& -9 = \\

(-35) ÷ (+7) = & -8 \\
& 2 = 
\end{array}
\]
Try on your own:
Solve each of the following

<table>
<thead>
<tr>
<th>36 [\div] 3</th>
<th>-100 [\div] -2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+27) [\div] (+9)</td>
<td>-19</td>
</tr>
</tbody>
</table>
Class Examples:
Solve each of the following:

It is currently $-4^\circ C$ in Calgary at 6:00am. The temperature will rise $2^\circ C$ every hour until 2:00pm today. What will the temperature be at 10:00am?

Try on your own:
Solve each of the following:

Jordan currently has $90$ in his bank account. He wants to earn some more money so starts cutting lawns in his neighborhood. If he cuts 8 lawns this week and charges $20$ each. What will he have in his bank account after cutting these lawns?

CARD GAME: Salute
MATH STACKS: Integer Review
**INTRODUCTION**

**Integers** - A set of positive and negative whole numbers. They can be represented on a number line.

**THE NUMBER LINE**

<table>
<thead>
<tr>
<th>Negative Numbers</th>
<th>Positive Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>-9</td>
<td>9</td>
</tr>
<tr>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>-7</td>
<td>7</td>
</tr>
<tr>
<td>-6</td>
<td>6</td>
</tr>
<tr>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Absolute Value** - The distance a number is from zero on the number line. An absolute value is never negative. Examples: \(|-5| = 5\) and \(|5| = 5\)

---

**ADDING INTEGERS**

**SAME SIGN** - Add and Keep the Sign!

Add the absolute value of the numbers and keep the same sign.

\[(\text{positive}) + (\text{positive}) = \text{Positive}\]
\[(+4) + (+5) = +9\]

\[(\text{negative}) + (\text{negative}) = \text{Negative}\]
\[(-4) + (-5) = -9\]

**DIFFERENT SIGNS** - Subtract and Keep the Sign of the Bigger Number!

Subtract the absolute value of the numbers and keep the sign of the bigger number.

\[(-4) + (+5) = +1\]
\[(+4) + (-5) = -1\]

---

**SUBTRACTING INTEGERS**

Do not subtract integers. You must change the signs: "Add the Opposite"

**KEEP** - Keep the sign of the first number

**CHANGE** - Change the subtraction sign to addition

\[(+4) - (-4)\]

Keep change change
\[(+4) + (+4)\]

**NOW USE THE RULES FOR ADDING:**

**SAME SIGN** - Add absolute values and keep sign:

\[(+4) + (+4) = 8\]

---

**MULTIPLYING INTEGERS**

**SAME SIGNS** - POSITIVE

Multiply the numbers. Answer will be positive.

\[(-5) \times (-5) = +25\]

**DIFFERENT SIGNS** - NEGATIVE

Multiply the numbers. Answer will be negative.

\[(+5) \times (-5) = -25\]

---

**DIVIDING INTEGERS**

**SAME SIGNS** - POSITIVE

Divide the numbers. Answer will be positive.

\[(-5) \div (-5) = +1\]

**DIFFERENT SIGNS** - NEGATIVE

Divide the numbers. Answer will be negative.

\[(+5) \div (-5) = -1\]
Operations of Integers

1. Add the following:
   a. \((+17) + (-11) = \) ________
   d. \((-41) + (+2) = \) ________
   b. \((-18) + (-11) = \) ________
   e. \((-22) + (+12) = \) ________
   c. \((-10) + (+11) = \) ________
   f. \((-11) + (-3) = \) ________

2. Subtract the following.
   a. \((+25) - (+25) = \) ________
   d. \((-2) - (-3) = \) ________
   b. \((-16) - (-11) = \) ________
   e. \((-7) - (+5) = \) ________
   c. \((+22) - (+11) = \) ________
   f. \((+7) - (+10) = \) ________

3. Multiply the following.
   a. \((+7)(+9) = \) ________
   d. \((-20)(0) = \) ________
   b. \((-4)(-5) = \) ________
   e. \((+7)(-6) = \) ________
   c. \((-3)(+5) = \) ________
   f. \((+3)(-6) = \) ________

4. Divide the following.
   a. \((24) \div (-12) = \) ________
   d. \((+100) \div (+2) = \) ________
   b. \((-16) \div (-2) = \) ________
   e. \((-48) \div (-12) = \) ________
   c. \((-6) \div (+2) = \) ________
   f. \((-36) \div (+9) = \) ________
5. Combine the following integers:
   a. \((+5) + (-3) + (-2) = \) __________
   d. \((-3) \times (-4)(0) = \) __________

   b. \((0) + (-3) + (-7) = \) __________
   e. \((-3) \times (-2)(-3)(+1)(-1) = \) __________

   c. \((-4) + (-4) + (+4) = \) __________
   f. \((8) \div (2)(-3) = \) __________

Problem Solving

1. The temperature in Resolute, Nunavut, one afternoon in May was \(-8^\circ C\). The temperature decreased by \(6^\circ C\) to reach the overnight low temperature. What was the overnight low temperature?

2. The element mercury is a silver-colored liquid at room temperature. The melting point of mercury is \(-39^\circ C\). The boiling point of mercury is \(357^\circ C\). How many degrees is the boiling point above the melting point?

3. Ana owns 75 shares of the Leafy Greens Company. One week, the value of each share dropped by 60 cents. The next week, the value of each share grew by 85 cents. What was the total change in the value of Ana’s shares
   a. in the first week?
   b. the second week?
   c. over the two-week period?

4. From 11:00 p.m. to 5:00 a.m., the temperature in Saskatoon fell from \(-1^\circ C\) to \(-19^\circ C\).
   a. What was the change in temperature?
   b. What was the change in temperature per hour? What assumption did you make?
Unit 1: Number Sense
Lesson 3

Exponents & Exponent Laws

Concepts
- Introduce exponents in both expanded and standard form including all corresponding definitions.
- Explore all Exponent Laws; Multiplication, Division, Power of a Power, Zero Property & Negative exponents.

Self-Check
- I can represent a power in expanded form and/or standard form.
- I can combine powers using multiplication law
- I can combine powers using division law
- I can combine powers using power or a power law
- I can convert positive exponents to negative exponents

Definitions:
Exponents are a way of expressing repeated multiplication.

The base is the number to be repeated, the exponent tells us how many times it is to be repeated.

Together, they make a power.

Exponential form is a number written as a power (with base and exponent).

The expanded form of the power above is $6 \times 6 \times 6 \times 6 \times 6$.
The base of 6, is multiplied by itself 5 times.

The standard form of a number is the answer of the repeated multiplication sentence. $6 \times 6 \times 6 \times 6 \times 6 = 7776$.
7 776 is the standard form of this number.
Complete the following chart:

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Expanded Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^5$</td>
<td>$6 \times 6 \times 6 \times 6 \times 6$</td>
<td>7,776</td>
</tr>
</tbody>
</table>

**CLASS EXAMPLES**

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Expanded Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.6^2$</td>
<td>$7 \times 7 \times 7$</td>
<td>27</td>
</tr>
</tbody>
</table>

**TRY ON YOUR OWN**

<table>
<thead>
<tr>
<th>Exponential Form</th>
<th>Expanded Form</th>
<th>Standard Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8^3$</td>
<td>$3 \times 3 \times 3 \times 3 \times 3$</td>
<td>625</td>
</tr>
</tbody>
</table>

**Negative Powers and Negative Bases**

Examine these two powers.

$$(-3)^4 \quad \text{and} \quad -3^4$$

$(-3)^4$ is an example of a **negative base**; which means the negative sign should be repeated.

The expanded form of this number is $(-3) \times (-3) \times (-3) \times (-3)$ with a product of 81.

$-3^4$ is an example of a negative power.

In this scenario, the exponent applies only to the base of 3 and therefore the negative sign is **NOT** repeated. The negative means the entire power will be negative.

The expanded form of this number is $-(3 \times 3 \times 3 \times 3)$ with a product of $-81$.

81 & $-81$ are two very different products.
Exponent Laws

1st Law: Multiplying Powers with the Same Base

\[ a^n \times a^m = a^{n+m} \]

When multiplying powers with the same base, you keep the base and add the exponents.

**Example:**
\[ 2^2 \times 2^3 \]
\[ 2^2 \cdot 3 \]
\[ 2^5 \quad \text{Exponential form} \]
\[ 32 \quad \text{Standard Form} \]

**Example:**
\[ \left( \frac{1}{4} \right)^4 \times \left( \frac{1}{4} \right)^2 \]
\[ \left( \frac{1}{4} \right)^{4+2} \]
\[ \left( \frac{1}{4} \right)^6 \quad \text{Exponential form} \]
\[ \frac{1}{4096} \quad \text{Standard Form} \]

**Example:**
\[ 2^1 \times 3^2 \]
\[ 2^1 \cdot 3^2 \]
It cannot be simplified because the bases are not the same. The **product law** does not apply.

Class Examples

| \[ 5^3 \times 5^2 \] | \[ \left( \frac{1}{3} \right)^2 \times \left( \frac{1}{3} \right)^2 \] |
| \[ 7^0 \times 7^3 \] | \[ x^5 \times x^2 \] |

Try on your own

| \[ 9 \times 9^3 \] | \[ 4 \times 4^2 \times 4^3 \] |
| \[ \left( \frac{2}{5} \right)^1 \times \left( \frac{2}{5} \right)^1 \] | \[ (a^4)(a^3) \] |
2nd Law: Dividing Powers with the Same Base

\[ a^n \div a^m = a^{n-m} \quad m \neq 0 \]

When dividing powers with the same base, you can keep the base and subtract the exponents.

Example: \(5^4 \div 5\)

\[ \frac{5^4}{5^1} = 5^{4-1} \]

\[ 5^3 \quad \text{Exponential form} \]

\[ 125 \quad \text{Standard Form} \]

Class Examples

<table>
<thead>
<tr>
<th>(3^4 \div 3^2)</th>
<th>(5^7 \div 5^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3^2)</td>
<td>(5^{7-3})</td>
</tr>
<tr>
<td></td>
<td>(5^4)</td>
</tr>
</tbody>
</table>

Try on your own:
Simplify and evaluate the following.

<table>
<thead>
<tr>
<th>(10^6 \div 10)</th>
<th>(4^1 \div 4^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^5)</td>
<td>(4^1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(6^3 \div 6^2)</th>
<th>(m^8 \div m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6^1)</td>
<td>(m^{8-2})</td>
</tr>
</tbody>
</table>

3rd Law: Power of a Power

\[(a^n)^m = a^{a \times m}\]

\[(2a^n)^m = 2^m \times a^{n \times m}\]

When you have a power that has another exponent (known as a Power of a Power), you can keep the base and multiply the exponents.

Example: \((5^4)^2\)

\[5^4 \times 2\]

\[5^8\]

\[(5x)^2\]

\[5^2 \times x^2\]

\[25x^2\]

\[(8^2 \times 4^5)^3\]

\[8^{2 \times 3} \times 4^{5 \times 3}\]

\[8^6 \times 4^{15}\]
Class Examples:
Simplify the following.

<table>
<thead>
<tr>
<th>$\left(3^8\right)^3$</th>
<th>$\left(4^0\right)^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(5x^3)^3$</td>
<td>$(b^2)^{10}$</td>
</tr>
</tbody>
</table>

Try on your own:
Simplify the following.

<table>
<thead>
<tr>
<th>$(7^3)^4$</th>
<th>$(18^5)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2^4)^4$</td>
<td>$(4w^2)^2$</td>
</tr>
</tbody>
</table>

4th Law: Power of a Quotient

\[
\frac{n^a}{m^a} = \left(\frac{n}{m}\right)^a
\]

When you divide powers with different bases but the same exponents, divide the bases, and raise the quotient to the common exponent.

Example:

\[
\frac{12^3}{4^3} \quad \frac{(m^2)^2}{3^2} \quad \frac{m^{2\times2}}{3^2} \quad \frac{m^4}{9}
\]

Class Examples:
Simplify the following.

<table>
<thead>
<tr>
<th>$\left(\frac{25^3}{5^3}\right)$</th>
<th>$\left(\frac{3}{2x}\right)^2$</th>
</tr>
</thead>
</table>
### Try on your own
Simplify the following.

| \[
\left( \frac{16^{-5}}{20^{-5}} \right) \]
| \[
\left( \frac{a^3}{8} \right)
\]

---

### 5th Law: Powers with Exponents of ZERO

\[ a^0 = 1 \quad \text{Note: } a \neq 0 \]

Any non-zero base raised to an exponent of zero equals one.

**Example:**

\[
\begin{align*}
2^3 & \div 2^3 \\
2^3 & \div 2^3 \\
2^3 & \div -3 \\
2^0 & \\
1
\end{align*}
\]

\[
\begin{align*}
(0.7)^4 & \times (0.7^{-4}) \\
(0.7)^4 & \div (-4) \\
(0.7)^0 & \\
1
\end{align*}
\]

**Class Examples:**
Simplify and evaluate the following.

| \[5^2 \div 5^2\] | \[2^8 \times 2^{-8}\] |

---

### Try on your own:
Simplify and evaluate the following.

| \[(0.1)^2 \times (0.1)^2\] | \[
\left( \frac{2}{5} \right)^1 \div \left( \frac{2}{5} \right)^1
\]

| \[
\left( \frac{1}{3} \right)^{-3} \times \left( \frac{1}{3} \right)^3
\] | \[
\left( \frac{1}{4} \right)^{-3} \div \left( \frac{1}{4} \right)^{-3}
\] |
6th Law: Powers with Negative Exponents

\[ a^{-n} = \frac{1}{a^n} \]

To simplify any non-zero base with a negative exponent, the exponent must become positive. When a power has a negative exponent, take the reciprocal of the base and raise it to a positive exponent.

**NOTE:** Reciprocal of a number means to interchange the numerator and denominator. For example, the reciprocal of 3 is \( \frac{1}{3} \).

**Example:**

\[
\begin{align*}
(2)^{-1} &= \left(\frac{1}{2}\right)^1 &= \frac{1}{2} \\
(3)^{-4} &= \left(\frac{1}{3}\right)^4 &= \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}
\end{align*}
\]

**Class Examples:**
Simplify and record your answer using positive exponents.

<table>
<thead>
<tr>
<th>((4)^2)</th>
<th>(\left(\frac{1}{3}\right)^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Try on your own:**
Simplify and record your answer using positive exponents.

<table>
<thead>
<tr>
<th>((\frac{4}{5})^{-1})</th>
<th>(\left(\frac{2}{3}\right)^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simplifying Expressions:

- To **simplify** an exponential expression means to use the exponent laws & write the expression as a single power.
- Simplified expressions cannot have any negative exponents.
- To **evaluate** a power means to find the numeric value of the simplified exponential expression.

**Example:**

\[
7^4 \div 7^{-2} \times 7^{-3} = \frac{(7)^4}{(7)^{-2}} \times 7^{-3} = 7^{6} \times 7^{-3} = 7^3. \\
3^{-3} \times 3^5 \times 3^{-4} = \frac{3^{-3}}{3^{-4}} = \frac{3^2}{3} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}.
\]

**Class Examples:**
Simplify and record your answer as a single power using positive exponents.

\[
(-3)^{-1} \div (-3)^{-2} \times (-3)^{-4} = (-3)^{-1-(-2-4)} = (-3)^{1}. \\
\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2.
\]

**Try on your own:**
Simplify and record your answer as a single power using positive exponents.

\[
\frac{2^6 \times 2^{-3}}{2^{-4}} = \frac{2^6}{2^{-3}} \times 2^4 = 2^{6-(-3)} \times 2^4 = 2^9. \\
\left(\frac{7^{-1}}{7^{-2}}\right)^2 = \left(\frac{1}{7}\right)^{-1} = 7.
\]

**CARD GAME:**
Exponent War
Exponent Color by Number
1. Evaluate:
   a) \((-3)^2\) 
   b) \((2 + 3)^0\)
   c) \(8^8 ÷ 8^2\)
   d) \((4)^2 \times (4^2)^3\)
   e) \(4 - (3 \times 2)^2\)

2. Simplify. Leave your answer in exponential form.
   a) \(3^4 \times 2^3 \times 3^2 \times 2\)
   b) \(\frac{4^3 \times 4^5}{4^2}\)
   c) \(3^0 \times 3^1 \times 3^2\)
   d) \(w^4 \times w^7\)

3. Simplify. Leave your answer in exponential form using only positive exponents.
   a) \(3^2 \times 5^{-2}\)
   b) \(\frac{x^6 z^7}{y^{-3}}\)
   c) \(2^4 ÷ 2^6 \times 2^1\)
   d) \(5^2 \times x^3 \times 5 \times x\)
4. Simplify & write in exponential form:
   a) \((3^2)^4\)  
   b) \([(3^2)^4]^3\)  
   c) \(((−5)^3)^3\)  
   d) \((2^5)^6\)  
   e) \([(-2)^2]^3\)  
   f) \([(-4)^3]^6\)

5. Evaluate:
   a) \((2^5 \div 2^3)^2\)  
   b) \((1 + 2)^3 - (5 - 2)^2\)  
   c) \((2^3 \times 2)^2\)  
   d) \((2^2)^3 - (2^2 + 1)^2\)

6. Use <, > or = to write a true sentence.
   a) \((-3)^5 \quad _____ \quad -3^5\)  
   b) \((-\frac{3}{4})^2 \quad _____ \quad (-\frac{3}{4})^4\)  
   c) \((-3)^2 \quad _____ \quad -(2)^4\)  
   d) \(2^5 \quad _____ \quad 5^2\)
Unit 1: Number Sense

Order of Operations

Concepts
- Explain and apply the order of operations using BEDMAS.

Self-Check
- I can recognize brackets and exponents
- I can solve questions involving multiplying and dividing
- I can solve questions involving addition and subtraction
- I can solve problems using the correct order of operations using BEDMAS
- I can solve error detection questions that require multiple order of operations

Order of Operations

Order of operations is an established order to follow when several operations must be performed.

1. Perform the operations inside the brackets first.
2. Evaluate any exponents.
3. Perform operations of division and multiplication in the order (left to right) they appear in an expression.
4. Perform operations of addition and subtraction in the order (left to right) they appear in an expression.

Note: It is very important to show all your steps and perform one operation per line.

You can remember the order of operations by using the acronym BEDMAS.
Class Example: \(5^2 - (17 - 3) + 8\)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5^2 - (17 - 3) + 8)</td>
<td>Brackets ((17 - 3 = 14))</td>
</tr>
<tr>
<td>(5^2 - 14 + 8)</td>
<td>Exponents ((5^2 = 5 \times 5))</td>
</tr>
<tr>
<td>(25 - 14 + 8)</td>
<td>Subtraction (it comes first when you go left to right)</td>
</tr>
<tr>
<td>(11 + 8)</td>
<td>Addition</td>
</tr>
<tr>
<td>(19)</td>
<td></td>
</tr>
</tbody>
</table>

Try on your own:

\((-3 + 6)^2 - 4(3)^2\)

Example: \(\frac{4^2 + 5^2 - 1}{3^2 - 1}\) (Always solve the numerator separate from solving the denominator)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{16 + 25 - 1}{9 - 1})</td>
<td>(4^2 = 16) \text{ and } (5^2 = 25) \text{ (exponents in numerator)} (3^2 = 9) \text{ (exponents in denominator)}</td>
</tr>
<tr>
<td>(\frac{40}{8})</td>
<td>(16 + 25 - 1 = 40) \text{ (} +/\text{- numerator} ) (9 - 1 = 8) \text{ (} +/\text{- denominator} )</td>
</tr>
<tr>
<td>(\frac{5}{5})</td>
<td>(\frac{40}{8} = 5) \text{ Divide Numerator and Denominator}</td>
</tr>
</tbody>
</table>

Try on your own:

\(\frac{3^2 \times [15 + (-3)]}{(+6) \times [(+11) + (-8)]}\)
Order of Operations

1. \( 4 \times 6 \times 3 \div 9 = \) __________

2. \( (15 - 9) \div (24 \div 8) = \) __________

3. \( (4 + 3 \times 2)^2 - 8 \times 2 \times 5 = \) __________

4. \( 5 \times 7 + 10 \times 2 \div 5 - 9 \times 2 = \) __________

5. \( 24 - 2^2 + (7^2 - 5^2) = \) __________

6. \( \frac{27 + 3 + 1}{2x + 2 + 1} = \) __________

7. Find the step where Justin made an error. Show the correct answer.

\[
32 \div (-2)^3 + 5 \times (4)^2
\]

\[
\begin{align*}
32 \div (-8) + 5 \times 8 & \quad \text{Step 1} \\
-4 + 5 \times 8 & \quad \text{Step 2} \\
-4 + 40 & \quad \text{Step 3} \\
36 & \quad \text{Step 4}
\end{align*}
\]

8. Adam has $450. On Monday he spends $210 on food. On Tuesday he divides the remaining money into four equal parts, he then spends three of those parts and keeps the 4\(^{th}\). Write an expression to represent this situation and determine the amount of money Adam has left.

9. Linda enters a book store and purchases the following: 3 notebooks for $1.20 each, a box of pencils for $1.50, and a box of pens for $1.70. How much did she spend at the bookstore?

CLASS ACTIVITY

CARD GAME: Got it, Closest to
**Unit 1: Number Sense**

**Factors & Products**

**Concepts**
- Understand the difference between prime & composite numbers
- Demonstrate an understanding of factors by completing prime factorizations
- Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.

**Self-Check**

- I can state the factors of a number
- I can represent a number as product of its factors
- I can complete a prime factorization & represent the factors using powers
- I can apply the divisibility rules to whole numbers

**Definitions:**

**Factors** are numbers that multiply together to produce a number.

For example, 2 and 6 are factors of 12 because $2 \times 6 = 12$.

**A Prime number** is a number whose only factors are one and itself.

For example, 13 is a prime number because its only factors are 1 and 13 (itself).

**Composite number** is a number that has more than two factors.

For example, 8 is a composite number because it has more than 2 factors. (1, 2, 4, 8).

**Class Examples:**

State of the factors of the following numbers:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

**Try on your own:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>
Activity: Sieve of Eratosthenes

1. Cross out the number 1.
2. Circle the number 2, then cross out all multiples of 2.
3. Circle the number 3, then cross out all multiples of 3.
4. Circle the number 5, then cross out all multiples of 5.
5. Circle the number 7, then cross out all multiples of 7.
6. Circle all remaining numbers.

Follow up Questions:

1. How many prime numbers are there between 1 and 100? ________

2. List the prime numbers from your completed chart.
   ________________________________________________________________________

3. What is the only even prime number? ____________
**Definition:**

*Prime Factorization:* writing a number as a product of its prime factors.

For example, \( 40 \rightarrow 2 \times 2 \times 2 \times 5 \rightarrow 2^3 \times 5 \)

**Steps:**

1. Write the number as a product of any two factors.
   * It doesn’t matter which two factors you start with.

2. Circle any prime numbers and continue to break down any composite numbers.
   Continue this process until all numbers are primes (all numbers are circled).

3. Write the number as a product of all its prime factors (the circled numbers) in order from smallest to biggest. Use exponents to combine factors that are the same.

   * **Note:** 1 is not used in factor trees.

**Example:** Write 20 in its prime factorization.

**Step One:** Write 20 as a product of any two factors. For example, \( 20 = 10 \times 2 \).

```
   20
   / \  \
 10  \  2
```

**Step Two:** Circle any prime numbers (2) and continue to breakdown any composite numbers (10).

```
   20
   /  \ \
 10  \  2
   / \  \
 2   5
```

**Step Three:** Write the product of all prime factors in order from smallest to biggest. Use exponents to combine common factors.

\[
20 = 2 \times 2 \times 5 = 2^2 \times 5
\]
Class Examples:
Write the prime factorization of the numbers below.

| 144 | 75 |

Try on your own:
Write the prime factorization of the numbers below.

| 76 | 121 |
1. Name a number that satisfies each of the descriptors below:
   a. An even prime
   b. A prime with identical digits
   c. The prime number between 47 and 59
   d. The largest prime number less than 100

2. I am a two-digit prime number. I am one less than a multiple of 10. If you add 30 to me you get another prime number. What number am I?

3. I have three different prime factors. I have two identical digits. What number am I?

4. Find the prime factorization of the following. Use a factor tree if it is useful.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 77</td>
<td>b. 95</td>
</tr>
<tr>
<td>c. 64</td>
<td>d. 135</td>
</tr>
<tr>
<td>e. 196</td>
<td>f. 200</td>
</tr>
<tr>
<td>g. 72</td>
<td>h. 120</td>
</tr>
</tbody>
</table>
Factors & Products: Part II

Divisibility Rules:

*Divisible* means that one number is a factor of the other (remainder of 0).

For example: 30 is divisible by 5, because $30 \div 5 = 6$

*Divisibility tests* are rules that are used to determine whether one number is divisible by another number.

### Divisibility Rules

<table>
<thead>
<tr>
<th>A number is divisible by. . .</th>
<th>Divisible</th>
<th>Not Divisible</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 if the last digit is even (0, 2, 4, 6, or 8).</td>
<td>3,978</td>
<td>4,975</td>
</tr>
<tr>
<td>3 if the sum of the digits is divisible by 3.</td>
<td>315</td>
<td>139</td>
</tr>
<tr>
<td>4 if the last two digits form a number divisible by 4.</td>
<td>8,512</td>
<td>7,518</td>
</tr>
<tr>
<td>5 if the last digit is 0 or 5.</td>
<td>14,975</td>
<td>10,978</td>
</tr>
<tr>
<td>6 if the number is divisible by both 2 and 3</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>9 if the sum of the digits is divisible by 9.</td>
<td>711</td>
<td>93</td>
</tr>
<tr>
<td>10 if the last digit is 0.</td>
<td>15,990</td>
<td>10,536</td>
</tr>
</tbody>
</table>

**Example:** Determine if the number 60 232 465 is divisible by 2, 3, 4, 5, 6, 9, or 10. Justify.

2? No; the last digit (5) is odd.
3? No; the sum of the digits (28) are not divisible by 3.
4? No; the last 2 digits (65) are not divisible by 4.
5? Yes; the last digit is 5.
6? No; the number is not divisible by 2 or 3.
9? No; the sum of the digits (28) are not divisible by 9.
10? No; the last digit is not a 0

60 232 465 is divisible by 5.
Class Example:
Determine if the following number is divisible by 2, 3, 4, 5, 6, 9, or 10. Justify your answers.

74 128

Try on your own:
Determine if the following numbers are divisible by 2, 3, 4, 5, 6, 9, or 10. Justify your answers.

92 245 7 641

Example: Create a five-digit number that is divisible by 4, 6, 9, and 10.
- To be divisible by 4, the last two digits must be divisible by 4.
- To be divisible by 6 the number must be even and the sum of the digits divisible by 3.
- To be divisible by 9, the sum of the digits must be divisible by 9.
- To be divisible by 10, the last digit must be 0.

Try on your own:
Create a 4-digit number divisible by 3 but not 9.

Class Example:
Create a 4-digit number divisible by 6 and 4.

Try on your own:
Create a 3-digit number divisible by 6 and 4.
1. Write the prime factors of the following numbers.
   a. 10
   b. 12
   c. 24
   d. 112
   e. 200
   f. 2016

2. Check off the numbers that each of the following is divisible by.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 840</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>234 675</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 212 452</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99 876</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48 560</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Create a 4-digit number divisible by 3 and 4.

4. Create a 5-digit number divisible by 6 but not by 9.
5. Find the missing digit in each of the following numbers to make each statement true.

a) The number $75\_4$ is divisible by 4.

b) The number $66\_9$ is divisible by 9.

c) The number $\_05$ is divisible by 5.

d) The number $\_98$ is divisible by 3.

e) The number $37\_\_\_\_$ is divisible by 6.
Greatest Common Factor (GCF) & Lowest Common Multiple (LCM)

Concepts
- Identify the multiples involving up to three numbers
- Demonstrate an understanding of the Lowest Common Multiple (LCM) & Greatest Common Factors (GCF)
- Recognize and write the factors for up to three numbers using prime factorization
- Able to determine the GCF & LCM using prime factorization method

Self-Check
- I can write multiples of a number
- I can find the factors of a number using prime factorization
- I can find the factors by using divisibility rules
- I can determine the Lowest Common Multiple for two different numbers
- I can determine the Greatest Common Factor for two different numbers

Definitions:

Multiples- is the product of two whole numbers in any given order.
For example, the multiples of 7 are 7, 14, 21, 28, 35, 42 …

Common Multiple - is a multiple shared by two numbers.
For example, 12 and 24 are two common multiples of 4 and 6.

Lowest Common Multiple (LCM) - is the smallest common multiple shared by two numbers.
For example, 60 is the lowest common multiple of 5 and 12.

Factors - are numbers that multiply to a given number.
For example, 2 and 6 are factors of 12 because $2 \times 6 = 12$.

Common Factors - are factors that are shared among numbers.
For example, 2 is a common factor to 10 and 6 because $2 \times 5 = 10$ and $2 \times 3 = 6$.

Greatest Common Factor (GCF) - is the largest factor that is common to two or more numbers.
For example, 4 is the greatest common factor to 8 and 12 because it is the largest number that divides into both numbers.
Class Example: **List Method**

Find the Lowest Common Multiple between 5 and 7:

Step 1: List the first few multiples of each number
- 5: 5, 10, 15, 20, 25, 35, 40, ...
- 8: 8, 16, 24, 32, 40, ...

Step 2: Locate the lowest multiple that both numbers have in common
- LCM (5 & 8) = 40

Class Example: **Prime Factorization Method**

Find the lowest common multiple for 16 and 40:

Step 1: Complete a prime factorization for each number.

```
16: 2 \cdot 2 \cdot 2 \cdot 2 \\
40: 2 \cdot 2 \cdot 2 \cdot 5
```

Step 2: Write the prime factorization as repeated multiplication.
- 16: \(2^4\)
- 40: \(2^3 \cdot 5\)

Step 3: Select the highest power for each number that exists in the repeated multiplication.
- \(LCM(16, 40) = 2^4 \cdot 5 = 80\)

**Class Examples:** Find the LCM.

<table>
<thead>
<tr>
<th>6 and 7</th>
<th>6 and 15</th>
</tr>
</thead>
</table>

**Try on your own:** Find the LCM.

| 12 and 15 | 4 and 11 |
**Class Example: List Method**

Find the Greatest Common Factor (GCF) 40 and 80:

Step 1: List all of the factors of both numbers.

<table>
<thead>
<tr>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

Factors of 40 are 1,2,4,5,8,10,20,40

Factors of 80 are 1,2,4,5,8,10,16,20,40

Step 2: Select the Greatest Common Factor.  
GCF (40,80) = 40

**Class Example: Prime Factorization Method**

Find the greatest common factor (GCF) for 64 and 80

Step 1: Complete a prime factorization for both numbers.

Prime Factors for 64: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

Prime Factors for 80: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^4 \times 5$

Step 2: Select the prime factors that both prime factorizations have in common.  
Their product is the GCF.

$GCF(64,80) = 2 \cdot 2 \cdot 2 = 16$
**Class Examples:** Find the GCF.

<table>
<thead>
<tr>
<th>14 and 16</th>
<th>18 and 27</th>
</tr>
</thead>
</table>

**Try on your own:** Find the GCF.

<table>
<thead>
<tr>
<th>100 and 30</th>
<th>20 and 16</th>
</tr>
</thead>
</table>
1. List four consecutive multiples of the following numbers.
   a) 4, _____, _____, _____, _____
   b) 10, _____, _____, _____, _____
   c) 5, _____, _____, _____, _____

2. Determine the lowest common multiple for each pair of numbers:
   a. 18 and 27          c. 16 and 24
   b. 32 and 40          d. 9, 12 and 18

3. Determine the greatest common factor for each pair of numbers:
   a. 28 and 49          c. 32 and 42
   b. 18 and 24          d. 28, 42 and 64

4. Find two different numbers which both have the following factor in common:
   a. 24                 b. 13

5. Explain the difference between determining the greatest common factor and the least common multiple of 12 and 14.
This review will cover:

Lesson 1: Number Systems  Lesson 2: Integers  
Lesson 3: Exponents  Lesson 4: Order of Operations  
Lesson 5: Factors & Products  Lesson 6: GCF & LCM

1. Place a check beside the number system(s) each of the following numbers belongs to:  (SO 1)

<table>
<thead>
<tr>
<th></th>
<th>Natural</th>
<th>Whole</th>
<th>Integer</th>
<th>Rational</th>
<th>Irrational</th>
</tr>
</thead>
<tbody>
<tr>
<td>−17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt{11})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Calculate the following:  (SO 2,3,4)

a) \((+3) + (−7) + (+8)\)  
f) \((-12)(−5) − (40)\)  

b) \((-35) − (+6) \times (+3)\)  
g) \(\frac{24}{3} \times 2 ÷ (−4)\)  

c) \(7^5 ÷ 7^{-3}\)  
h) \((-3)^2 \times 2 + 5\)  

d) \((4x^3y)^2\)  
i) \(\frac{5^2 + 2}{3}\)  

e) \(\frac{14x^5}{7x^7}\)  
j) \((5)^2 − [(+7)(+3) − (+6)(+5)]\)  

3. A small factory employs 8 workers. Of these, 4 receive a wage of $150 per day and the rest receive $85 per day. A week consists of 5 working days. How much does the factory pay out for salaries each week?
4. A submarine dives from the surface of the water at 15 \( m/min \) for 6 minutes. The engine is then turned off and the submarine floats upward at a rate of 2 \( m/min \) for 15 minutes. Where is the submarine in relation to the surface of the water after this 21 minutes? (SO 2, 4)

5. List the numbers less than 75 which can be divided by both 3 and 5. (SO 5)

6. Insert any operation signs ( ( ), +, -, \( \times \), \( \div \) ) so that the given numbers make the statement true. (SO 4)
   a) \( 2 \ 3 \ 5 \ 6 = 23 \)  
   b) \( 5 \ 6 \ 7 \ 9 = 64 \)

7. For each number below, list: i) all of the factors  
   ii) the first 3 multiples (SO 5)
   a) 36  
   b) 19  
   c) 15

8. Determine the GCF & LCM for each set of numbers.  
   a) 24 and 45  
   b) 12, 15 and 18 (SO 6)

9. Complete a prime factorization for the following numbers and write your solution as a product of its primes.  
   a) 54  
   b) 360 (SO 5)
Unit 1B: Number Sense
Introduction to Decimals

Concepts
- Compare and order positive fractions, positive decimals, and whole numbers by using benchmarks, place value, equivalent fractions, and/or decimals.
- Estimate the sum, product, or difference of two numbers using rounding techniques.

Self-Check
- I can round numbers to a given place value
- I can estimate operations using rounding techniques

Place Value

In our decimal number system, the value of a digit depends on its place, or position, in the number. Each place has a value of 10 times the place to its right.

<table>
<thead>
<tr>
<th>PLACE VALUE CHART</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILLIONS</td>
</tr>
<tr>
<td>MILLISECONDS</td>
</tr>
<tr>
<td>TRILLIONS</td>
</tr>
</tbody>
</table>

Rounding a number means to take the exact number and re-write it as an estimated value to the nearest indicated place.

Remember:

To round you always look at the digit to the right of the indicated place.

If that digit is 5 or more you **round that digit up** and replace all of the other digits to the right of it with zeros.

If that digit is less than 5 you **leave that digit alone** and replace all of the other digits to the right of it with zeros.
**Round & Replace with Zeros**

**Example:** Round 1476 to the nearest ten.
Since 7 is the placeholder for the tens position, circle it and look at the digit to the right of it (6). Since 6 is greater than 5, we round 7 up to 8 and replace all digits to the right of the 8 with zeros.

1 4\(\bigcirc\)6 becomes 1 480

**Class Examples:**
Round each number to the indicated value.

<table>
<thead>
<tr>
<th>1 264 783 to the nearest ten thousand</th>
<th>139 281 to the nearest hundred</th>
</tr>
</thead>
</table>

**Try on your own:**
Round each number to the indicated value.

<table>
<thead>
<tr>
<th>196 843 203 to the nearest million</th>
<th>399 642 to the nearest thousand</th>
</tr>
</thead>
</table>

**Rounding** is a useful tool to help us estimate. Estimation gives us an approximate answer, and can help us decide if our answer was reasonable or not. Let’s practice some estimating.

Example: 
\[
\begin{array}{c}
11 \\
+67 \\
\hline
106
\end{array}
\]
-----Rounds to---------\rightarrow \[10 + 70 = 80\] 

**Class Examples:**
Estimate the following:

<table>
<thead>
<tr>
<th>127+138 to the nearest ten.</th>
<th>139 -41 to the nearest ten.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2226 + 1001 to the nearest hundred.</td>
<td>7080 – 1678 to the nearest hundred.</td>
</tr>
</tbody>
</table>
**Rounding Decimals**

Using the place value chart, decide on which number will be left when we finish.

- Rounding to **tenths** means one decimal place (one number to the right of the decimal).
- Rounding to hundredths means two decimal places (two numbers to the right of the decimal).
- Etc.

After choosing the appropriate place value to round to, look to the right of that number. If it is 5 or higher, it will cause the selected place value to increase by 1.

---

**Example 1**

Round 15.67 to the nearest tenth.

\[
15.67 \quad \text{‘6’ is in the tenths place. We will be keeping one decimal place.}
\text{The 7 will cause the six to ‘round up’ to 7.}
\]

**Answer:** 15.7

---

**Example 2**

Round 0.054 to the nearest hundredth.

\[
0.054 \quad \text{‘5’ is in the hundredths place. We will be keeping two decimal places.}
\text{The 4 is below 5 and no rounding will happen.}
\]

**Answer:** 0.05

---

**Example 3**

Round 13.998 to the nearest tenth.

\[
13.986 \quad \text{‘9’ is in the tenths place. We will be keeping one decimal place.}
\text{The 8 will cause the 9 to round up to 10, because we reach 10, we need to increase the ones place by one.}
\text{This rounding has caused two numbers to change.}
\]

**Answer:** 14.0

It is most correct to keep the “.0”. When asked to round to the tenths, we should keep the place value as ‘0’.
### Introduction to Decimals

1. For each number, select the **place value** for the digit that is bold.

<table>
<thead>
<tr>
<th>Number</th>
<th>Ten Thousands</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
<th>Ten Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex. 56.778</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>121.335</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.298</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45 990.662</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33.33333</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 556.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Round the following number to the place value indicated. Replace with zeros if necessary.

<table>
<thead>
<tr>
<th>Given Number</th>
<th>Round to the nearest…..</th>
<th>Rounded Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: 23.6</td>
<td>Tens</td>
<td>20</td>
</tr>
<tr>
<td>Example: 249.556</td>
<td>Tenths</td>
<td>249.6</td>
</tr>
<tr>
<td>10.298</td>
<td>Hundredths</td>
<td></td>
</tr>
<tr>
<td>4 556.9</td>
<td>Thousands</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>Tens</td>
<td></td>
</tr>
<tr>
<td>878.95</td>
<td>Hundreds</td>
<td></td>
</tr>
<tr>
<td>15 678</td>
<td>Ten-thousands</td>
<td></td>
</tr>
<tr>
<td>0.0044</td>
<td>Thousandths</td>
<td></td>
</tr>
<tr>
<td>45.44</td>
<td>Ones</td>
<td></td>
</tr>
<tr>
<td>12.4456</td>
<td>Tens</td>
<td></td>
</tr>
<tr>
<td>342.97</td>
<td>Tenths</td>
<td></td>
</tr>
<tr>
<td>88.67</td>
<td>Hundreds</td>
<td></td>
</tr>
</tbody>
</table>

3. Circle the number that is closest to your given number.

a) Example: Given number: 45

   40 or 50
   0 or 100
   0 or 1000
b) Given number: 156

150 or 160

100 or 200

0 or 1000

c) Given number: 1722

1720 or 1730

1700 or 1800

1000 or 2000

0 or 10000
Concept:
- Identify and predict patterns when multiplying with decimals.
- Identify and predict patterns when dividing with decimals.
- Complete and correctly place the decimal in addition and subtraction.

Self – Check:
- I can identify and predict patterns when multiplying with decimals
- I can identify and predict patterns when dividing with decimals
- I can add and subtract with decimals

Adding & Subtracting Decimals:

Whenever a question requires the addition or subtraction of numbers with decimals follow these simple steps:

1. Line up the decimal points
2. Drag the decimal straight down
3. Add/Subtract as usual

Class Examples:
Solve each of the following:

\[ 4.31 + 3.1 = \quad 14.8 - 7.35 = \]

\[ 53.7 - 18.72 = \quad 14.923 + 1.834 = \]
Try on your Own:
Solve each of the following:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$39.5 - 7.99 = $</td>
<td>$14.923 + 1.85 =$</td>
</tr>
<tr>
<td>$45.98 + 8.23 = $</td>
<td>$49.7 - 8.6 =$</td>
</tr>
</tbody>
</table>

Multiplying and Dividing Decimals:

Complete the table for the given values:

<table>
<thead>
<tr>
<th></th>
<th>$\times10$</th>
<th>$\times100$</th>
<th>$\times1000$</th>
<th>$\times0.1$</th>
<th>$\times0.01$</th>
<th>$\times0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 3.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. 1250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What pattern do you see in the table above?

Complete the table for the given values:

<table>
<thead>
<tr>
<th></th>
<th>$\div10$</th>
<th>$\div100$</th>
<th>$\div1000$</th>
<th>$\div0.1$</th>
<th>$\div0.01$</th>
<th>$\div0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 3.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. 1250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What pattern do you see in the table above?

Try on your Own:
Complete e, f & g with numbers of your own choosing.
When we move beyond powers of 10, similar rules apply:

**MULTIPLICATION:**

The number of decimal places in the product should equal the total number of decimal places in the two factors being multiplied.

**Class Examples:**

a) State the location of the decimal point in the solution
b) Complete the multiplication

<table>
<thead>
<tr>
<th>46×1.8 =</th>
<th>3.4×0.24 =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3.14×12 =</th>
<th>1.02×0.72 =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Try on your Own:**

<table>
<thead>
<tr>
<th>2.6×8.9 =</th>
<th>2.96×80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DIVISION

Using the following questions create a rule (in your own words) to predict whether the quotient of a problem will be larger or smaller than the original number.

*You may use your calculator to solve.

a. $41.5 \div 8.3 =$

b. $6.25 \div 1.5 =$

b. $336.49 \div 4.1 =$

c. $139.32 \div 0.54 =$

c. $4.05 \div 0.75 =$

d. $10.12 \div 0.92 =$

d. $35.855 \div 5.05 =$

e. $16.328 \div 3.14 =$

e. $3.84 \div 1.28 =$

f. $87.6 \div 2.4 =$

RULE:
Decimals Operations

1. Add the following, show your work.
   a. $43.57 + 104.6 = $
   b. $15.36 + 29.23 = $

2. Subtract the following, show your work.
   a. $83.06 - 12.3 = $
   b. $32.3 - 12.72 = $

3. Multiply the following, show your work.
   a. $0.03 \times 6 = $
   b. $260 \times 3.01 = $
   
   b. $0.05 \times 0.16 = $
   d. $5.29 \times 1.3 = $

4. State whether the quotient of each problem will be a larger or smaller number and explain why/how you know this.
   a. $22 \div 3.5$
   b. $15 \div 0.8$
   c. $\frac{52}{2}$
5. Barb bought 3 equally priced CDs and a DVD priced at $9.95. She paid $54.92 in total. What was the exact cost of each CD?

6. Barb bought 3 kg of apples at $1.50 a kilogram and 2 kg of potatoes at $1.98 a kilogram. Calculate the total cost.

7. If 4.5 L cost $22.41 what is the cost of 1 L of paint?

8. The price of gold was quoted at $4.08. If the price rose $1.08, fell $0.55, rose $1.27, and then fell $0.39, what was the closing price?

9. The perimeter of a rectangle is 56.8 cm. The width is equal to 4.4 cm. The length of the rectangle is ________ cm.
Unit 1: Number Sense

Fractions, Decimals & Percents

Concepts
- Convert between fractions, decimals and percents.

Self-Check
- I can convert values from decimal form to fraction form.
- I can convert values from fraction form to decimal form.
- I can convert fractions & decimals to percentages.

Definitions:

A **fraction** is a number that names part of a whole or part of a set

- **Numerator**: top number of a fraction; represents the part of the whole or set

- **Denominator**: bottom number of a fraction; represents the whole or set

Example: \( \frac{3}{5} \) could represent “three out of five slices of one whole pizza”

- “three out of the five children were girls”

A **decimal** is a number that uses a decimal point followed by digits that show a value smaller than one

A **percent** is a ratio whose second term is 100; it means part per hundred
Decimal, Fraction & Percent Conversions:

- To change a **fraction to a decimal**: Divide. (Numerator ÷ denominator).
  
  \[ \frac{7}{8} = 7 ÷ 8 = 0.875 \]

- To change a **fraction to a percent**: First change the fraction to a decimal, then change the decimal to a percent (×100).
  
  \[ \frac{4}{10} = 0.40 = 40\% \]

- To change a **percent to a fraction**: Divide by 100, drop the percent sign. Reduce the fraction to lowest terms.
  
  \[ 85\% = \frac{85}{100} = \frac{17}{20} \]

- To change a **percent to a decimal**: Move the decimal two places to the left (÷ 100). Drop the percent sign.
  
  \[ 124\% = 1.24 \]

- To change a **decimal to a percent**: Move the decimal two places to the right (×100), Add a percent sign.
  
  \[ 0.92 = 92\% \]

- To change a **decimal to fraction**: Write the fraction like you say it. Then simplify if needed.
  
  \[ 0.7 = "seven tenths" \rightarrow \frac{7}{10} \]
Complete the graphic organizer below to help you remember your conversion rules!

Converting Rational Numbers

Fractions, Decimals, and Percents are just different ways to show the same values. Percent means “per hundred.” Fractions show division.
**Class Examples:**
Complete the table below:

<table>
<thead>
<tr>
<th>Fraction/ Mixed Number</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{2}{5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td>14%</td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td>110%</td>
</tr>
</tbody>
</table>

**Try on your own:**
Complete the table below:

<table>
<thead>
<tr>
<th>Fraction/ Mixed Number</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{2}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. ( \frac{11}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td>59%</td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td>0.8%</td>
</tr>
</tbody>
</table>
1. Complete the following table:
   *Leave fractions and mixed numbers in reduced form.*

<table>
<thead>
<tr>
<th>Fraction/ Mixed Number</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>f. 0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g. 0.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h. 2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i.</td>
<td></td>
<td>34%</td>
</tr>
<tr>
<td>j.</td>
<td></td>
<td>235%</td>
</tr>
<tr>
<td>k.</td>
<td></td>
<td>4%</td>
</tr>
<tr>
<td>l. ( \frac{9}{20} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m. ( \frac{5}{8} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n. 3( \frac{2}{5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o. 2.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p.</td>
<td></td>
<td>0.62%</td>
</tr>
<tr>
<td>q. ( \frac{3}{4} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r.</td>
<td></td>
<td>0.53</td>
</tr>
<tr>
<td>s.</td>
<td></td>
<td>3.12%</td>
</tr>
</tbody>
</table>
Equivalent Fraction

Concept:
- Identify equivalent fractions, using multiplication and division.
- Create equivalent fractions, using multiplication and division.

Self – Check:
- I can identify equivalent fractions, using multiplication and division
- I can create equivalent fractions, using multiplication and division

Equivalent fractions are fractions that look different but have the same value. For example, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions because they both represent one-half.

Creating Equivalent Fractions by Multiplying or Dividing:

Steps:
1. Multiply/Divide both the numerator and denominator by the same number.
2. The resulting fraction is equivalent to the original fraction.

Example: Make an equivalent fraction to $\frac{1}{5}$.

1. Multiply both the numerator and denominator by the same number.
   \[
   \frac{1 \times 2}{5 \times 2} = \frac{2}{10}
   \]
2. The resulting fraction is equivalent to the original fraction.
   $\frac{2}{10}$ is equivalent to $\frac{1}{5}$

Example: Make an equivalent fraction to $\frac{10}{25}$.

1. Divide both the numerator and denominator by the same number.
   \[
   \frac{10 \div 5}{25 \div 5} = \frac{2}{5}
   \]
2. The resulting fraction is equivalent to the original fraction.
   $\frac{2}{5}$ is equivalent to $\frac{10}{25}$
**Class Examples:**
Write two equivalent fractions (using either multiplication or division) for each fraction:

| \[
\frac{96}{18}\] | \[
\frac{2}{5}\] |

---

**Try on your own:**
Write two equivalent fractions (using either multiplication or division) for each fraction:

| \[
\frac{3}{4}\] | \[
\frac{75}{300}\] |
| \[
\frac{9}{24}\] | \[
\frac{2}{7}\] |
Completing Equivalent Fractions:

**Example:** Find the missing number in the fraction.

\[
\frac{3}{4} = \frac{12}{\phantom{0}}
\]

Steps:
1. Determine the number and operation used to move from 3 to 12 (the known values)

\[
3 \times 4 = 12,
\]

2. Multiply the denominator by 4 to find the missing number.

The missing number is 16.

**Class Examples:**
Find the missing number for each of the following. Show your work.

\[
\frac{2}{3} = \frac{\phantom{0}}{39}
\]

\[
\frac{98}{\phantom{0}} = \frac{7}{14}
\]

**Try on your own:**
Find the missing number for each of the following. Show your work.

\[
\frac{4}{5} = \frac{76}{\phantom{0}}
\]

\[
\frac{\phantom{0}}{13} = \frac{39}{169}
\]
Simplifying Fractions

*Simplest form* or *lowest terms*

means the numerator and denominator of a fraction have no common factors other than 1.

For example, $\frac{2}{3}$ is in simplest form because 2 and 3 have no common factors other than 1.

For example, $\frac{6}{10}$ is not in simplest form because 6 and 10 have a common factor of 2.

Steps:
1. List out the factors of both the numerator and denominator.
2. Find the GCF of the numerator and denominator.
3. Divide both the numerator and the denominator by the GCF.

**Example:** Express $\frac{9}{12}$ in simplest form.

**Step One:** List out the factors of both the numerator and denominator.
- factors of 9: 1, 3, 9
- factors of 12: 1, 2, 3, 4, 6, 12

**Step Two:** Find the GCF of the numerator and denominator. GCF (9, 12) = 3

**Step Three:** Divide both the numerator and the denominator by the GCF.

\[
\frac{9}{12} \div \frac{3}{3} = \frac{3}{4}
\]

\[
\frac{3}{4} \text{ is in simplest form.}
\]

**Class Examples:**
Write each of the following fractions in simplest form. Show your work.

<table>
<thead>
<tr>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>42</td>
</tr>
</tbody>
</table>

**Try on your own:**
Write each of the following fractions in simplest form. Show your work.

<table>
<thead>
<tr>
<th>27</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
<td>40</td>
</tr>
</tbody>
</table>
Equivalent Fractions

1. Find two equivalent fractions for each of the following.
   a. \( \frac{5}{7} \)
   b. \( \frac{1}{2} \)
   c. \( \frac{30}{45} \)
   d. \( \frac{12}{36} \)

2. Find the missing number for each of the following.
   a. \( \frac{85}{\phantom{0}} = \frac{5}{6} \)
   b. \( \frac{128}{160} = \frac{8}{\phantom{0}} \)
   c. \( \frac{4}{\phantom{0}} = \frac{1}{6} \)
   d. \( \frac{12}{\phantom{0}} = \frac{\phantom{0}}{80} \)
   e. \( \frac{120}{180} = \frac{\phantom{0}}{15} \)
   f. \( \frac{108}{132} = \frac{\phantom{0}}{11} \)
   g. \( \frac{11}{\phantom{0}} = \frac{\phantom{0}}{126} \)
   h. \( \frac{6}{9} = \frac{\phantom{0}}{\phantom{0}} \)
   i. \( \frac{9}{\phantom{0}} = \frac{3}{4} \)
   j. \( \frac{\phantom{0}}{135} = \frac{5}{9} \)
3. Reduce the following fractions to lowest terms.

a. \( \frac{38}{42} = \)

i. \( \frac{3}{15} = \)

b. \( \frac{55}{110} = \)

j. \( \frac{18}{48} = \)

c. \( \frac{14}{32} = \)

k. \( \frac{8}{26} = \)

d. \( \frac{10}{37} = \)

l. \( \frac{10}{90} = \)

e. \( \frac{18}{24} = \)

m. \( \frac{108}{132} = \)

f. \( \frac{20}{28} = \)

n. \( \frac{9}{43} = \)

g. \( \frac{72}{80} = \)

o. \( \frac{3}{24} = \)

h. \( \frac{16}{24} = \)

p. \( \frac{50}{60} = \)

CARD & DICE GAMES:
Fraction War
Order in the Court
ROW GAME

CLASS ACTIVITY
**Unit 1: Number Sense**  
**Lesson 11**

**Fraction Operations: Add & Subtract Fractions**

**Concepts**
- Adding and subtracting fractions pictorially and symbolically.

**Self-Check**
- I can add fractions pictorially.
- I can add fractions symbolically (using numbers).
- I can subtract fractions pictorially.
- I can subtract fractions symbolically (using numbers).

---

**Adding & Subtracting Fractions Pictorially**

**Steps:**
1. Make a grid:
   - denominator of fraction 1 = number of rows (across)
   - denominator of fraction 2 = number of columns (up and down)

2. Model fraction 1 (numerator = the number of rows to shade in).
   Model fraction 2 (numerator = the number of columns to shade in).

3. Combine the individual squares of each fraction onto a common grid using the appropriate operation (+ or −).
   (Use more than one grid if necessary.)

4. State final answer.
   - Numerator = total # of squares shaded.
   - Denominator = total # of squares in the grid.

5. Write the fraction in lowest terms (divide numerator and denominator by the GCF).

**Example:**
\[
\frac{1}{3} + \frac{1}{4}
\]

**Step One:**
Make a grid: denominator of first fraction is 3 = THREE rows across  
denominator of second fraction 4 = FOUR columns down

3 by 4 grid

**Step Two:**
Model fraction 1 - \(\frac{1}{3}\) means shade in 1 of 3 rows.  
Model fraction 2 - \(\frac{1}{4}\) means shade 1 of 4 columns.

\[
\frac{1}{3} \quad \frac{1}{4}
\]
Step Three: Combine the individual squares of each fraction onto a common grid. (Use more than one grid if necessary.)

\[
\begin{array}{c|c|c|c|c|c}
\hline
& & & & & \\
\hline
& & & & & \\
\hline
& & & & & \\
\hline
\end{array}
\]

\[3 + 4 = 7\]

Step Four: State final answer.
Numerator = total # of squares shaded.
Denominator = total # of squares in the grid.

\[
\frac{1}{3} + \frac{1}{4} = \frac{7}{12}
\]

7 out of the 12 squares are shaded

Step Five: Write the fraction in lowest terms (divide numerator and denominator by the GCF).

Doesn’t apply for \(\frac{7}{12}\), already in lowest terms.

Class Examples:
Add/Subtract the following pictorially.

\[
\begin{array}{c}
\frac{3}{5} + \frac{2}{3} = \_\_\_\_\_\_\_\_
\\
\frac{1}{2} + 2\frac{2}{3} = \_\_\_\_\_\_\_\_
\\
\frac{1}{2} - \frac{1}{4} = \_\_\_\_\_\_\_\_
\\
2\frac{1}{2} - 1\frac{1}{7} = \_\_\_\_\_\_\_\_
\end{array}
\]
Try on your own:
Add/Subtract the following fractions pictorially.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{5} + \frac{2}{7})</td>
<td></td>
</tr>
<tr>
<td>(\frac{3}{4} + \frac{2}{3})</td>
<td></td>
</tr>
<tr>
<td>(2\frac{1}{2} + \frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{3} + 1\frac{2}{5})</td>
<td></td>
</tr>
<tr>
<td>(\frac{4}{5} - \frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(1\frac{4}{7} - 1\frac{1}{3})</td>
<td></td>
</tr>
</tbody>
</table>
Adding & Subtracting Fractions Symbolically (with Numbers)

Steps:
1. Write any mixed numbers as improper fractions.
2. Change each fraction to an equivalent fraction with the LCM as the denominator.
3. Add/Subtract the numerators and place the sum over the common denominator.
4. Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

Example: \( \frac{1}{4} + \frac{2}{5} \)

Step One: Write any mixed numbers as improper fractions.
Not required for this question.

Step Two: Change each fraction to an equivalent fraction with the LCM as the denominator.
\( \text{LCM}(4, 5) = 20 \)

\[
\left( \frac{1 \times 5}{4 \times 5} = \frac{5}{20} \right) \quad \left( \frac{2 \times 4}{5 \times 4} = \frac{8}{20} \right)
\]

Step Three: Add/Subtract the numerators and place the sum over the common denominator.
\[
\frac{5}{20} + \frac{8}{20} = \frac{13}{20}
\]

Step Four: Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

13 and 20 have no common factors.

\[
\frac{1}{4} + \frac{2}{5} = \frac{13}{20}
\]

Class Examples:
Add the following.

\[
\begin{align*}
\frac{2}{3} + & \frac{1}{5} \\
\frac{3}{8} + & \frac{5}{6}
\end{align*}
\]
### Class Examples:
Subtract the following.

\[
\begin{align*}
3 \frac{1}{4} - 2 \frac{2}{7} & \quad 3 \frac{4}{5} - 2 \frac{1}{15}
\end{align*}
\]

### Try on your own:
Add/Subtract the following:

\[
\begin{align*}
\frac{2}{9} + \frac{5}{6} & \quad \frac{3}{4} - \frac{7}{10} \\
\frac{4}{7} + 3 \frac{2}{5} & \quad 3 \frac{1}{2} - 2 \frac{4}{10}
\end{align*}
\]

### Applications:
A plumber cuts three pieces of pipe; \(4 \frac{2}{3} \text{ m}, \ 5 \frac{1}{5} \text{ m}, \) and \(3 \frac{7}{10} \text{ m}. \) What is the total length of the pipes?
### Fraction Operations: Add & Subtract

1. Add the following. Show your work.

<table>
<thead>
<tr>
<th>a. $4 + \frac{6}{10}$</th>
<th>b. $\frac{5}{14} + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. $\frac{4}{7} + \frac{2}{4}$</td>
<td>d. $\frac{7}{9} + \frac{2}{6}$</td>
</tr>
<tr>
<td>e. $2\frac{3}{8} + \frac{1}{9}$</td>
<td>f. $3\frac{2}{3} + \frac{5}{8}$</td>
</tr>
</tbody>
</table>
2. Subtract the following. Show your work.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$5 - \frac{4}{9}$</td>
</tr>
<tr>
<td>c.</td>
<td>$1\frac{1}{2} - \frac{1}{4}$</td>
</tr>
<tr>
<td>e.</td>
<td>$12\frac{2}{3} - 5\frac{1}{4}$</td>
</tr>
</tbody>
</table>
2. Leanne is learning a new dance routine. On Saturday, she rehearsed for $1\frac{2}{3}$ hours. On Sunday, she rehearsed for $2\frac{1}{4}$ hours. How long did Leanne spend rehearsing this weekend?

3. After $2\frac{2}{5}$ hours, a baseball game was almost over when the score became tied. The extra innings extended the total playing time to $4\frac{1}{4}$ hours. How long did the extra innings take?

5. Gerry and Raj operate cement-mixing trucks. On Monday morning, they each had a full truck of cement. By lunch time, Gerry had poured $\frac{7}{16}$ of his truck’s cement, and Raj had poured $\frac{5}{8}$ of his truck’s cement.

   a) Did they pour more or less than a total of one full truck of cement before stopping for lunch?

   b) How much cement did each person have in his truck at lunch time?
Unit 1: Number Sense

Lesson 11

Fraction Operations: Multiply & Divide

Concepts:
• Multiplying and dividing fractions pictorially and symbolically.

Self-Check:
☐ I can multiply fractions pictorially.
☐ I can multiply fractions symbolically (with numbers).
☐ I can divide fractions symbolically (with numbers).

### Multiplying Fractions Pictorially:

**Steps:**
1. Make a grid: denominator of fraction 1 = number of rows (across) 
   denominator of fraction 2 = number of columns (up and down)
2. Model fraction 1 (numerator = the number of rows to shade in) and fraction 2 
   (numerator = the number of columns to shade in) on the same grid.
3. State final answer.
   Numerator = total # of shaded squares that **overlap**.
   Denominator = total # of squares in the grid.
4. Write the fraction in lowest terms (divide numerator and denominator by the GCF).

**Example:** \(\frac{2}{3} \times \frac{1}{2}\)

**Step One:** Make a grid: denominator of fraction 1 = THREE means 3 rows across 
   denominator of fraction 2 = TWO means 2 columns down

\[ \begin{array}{|c|c|} \hline & \hline \end{array} \]

3 rows and 2 columns

**Step Two:** Model: fraction 1 (2 of the 3 rows need to be shaded) 
   fraction 2 (1 of the 2 columns need to be shaded) on the same grid.

\[ \begin{array}{|c|c|} \hline & \hline \end{array} \]

**Step Three:** State final answer.
   Numerator = total # of shaded squares that **overlap**.
   Denominator = total # of squares in the grid.

\[ \frac{2}{3} \times \frac{1}{2} = \frac{2}{6} \]

2 of the 6 squares overlap

**Step Four:** Write the fraction in lowest terms (divide numerator and denominator by the GCF).

\[ \frac{2}{6} ÷ 2 = \frac{1}{3} \]
**Class Examples:**
Multiply the following pictorially.

\[
\frac{3}{4} \times \frac{4}{3} = \quad \quad 1 \frac{2}{5} \times \frac{3}{4} = 
\]

**Try on your own:**
Multiply the following pictorially.

\[
\frac{2}{3} \times \frac{3}{5} = \quad \quad \frac{3}{7} \times \frac{2}{5} = \\
\frac{2}{3} \times \frac{1}{2} = \quad \quad \frac{2}{7} \times \frac{4}{5} = 
\]
Multiplying Fractions Symbolically

Steps: 1. Write any mixed numbers as improper fractions.
2. Multiply the numerators together and multiply the denominators together and place together as one fraction.
3. Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

Note: You do not have to have a common denominator to multiply fractions.

Example: \( \frac{1}{9} \times \frac{2}{5} \)

Step One: Write any mixed numbers as improper fractions.

\( \frac{10}{9} \times \frac{2}{5} \)

Step Two: Multiply the numerators together and multiply the denominators together and place together as one fraction.

\( \frac{20}{45} \)

Step Three: Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

\( \frac{20 \div 5}{45 \div 5} = \frac{4}{9} \)

Class Examples:
Multiply the following.

\( 7 \times \frac{3}{5} \)
\( 3\frac{1}{2} \times 2\frac{4}{10} \)

Note: All whole numbers can be written as fractions with a denominator of 1.

Try on your own:
Multiply the following.

\( \frac{5}{6} \times \frac{2}{8} \)
\( 3\frac{1}{4} \times 2 \)

\( \frac{3}{4} \times \frac{4}{3} \)
\( 3\frac{1}{5} \times 2\frac{1}{3} \)
Dividing Fractions

**Definition:**

Reciprocal is two numbers whose product is 1. It means to switch the numerator and the denominator.

For example, the reciprocal of \( \frac{3}{5} \) is \( \frac{5}{3} \).

**Steps:**

1. Write any mixed numbers as improper fractions.
2. Multiply the first fraction by the reciprocal of the second fraction.
3. Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

**Note:** You do not have to have a common denominator to divide fractions.

**Example:**

\[ \frac{2}{3} \div \frac{2}{5} \]

**Step One:** Write any mixed numbers as improper fractions.

\[ \frac{5}{3} \div \frac{2}{5} \]

**Step Two:** Multiply the first fraction by the reciprocal of the second.

\[ \frac{5}{3} \times \frac{5}{2} \]

\[ \frac{25}{15} \]

**Step Three:** Write the fraction in lowest terms (if possible) by dividing both the numerator and denominator by the GCF.

\[ \frac{25 \div 5}{15 \div 5} = \frac{5}{3} \]

Change the improper fraction to a mixed number.

\[ \frac{5}{3} = 1 \frac{2}{3} \]

\[ 1 \frac{2}{3} \div \frac{2}{5} = 1 \frac{2}{3} \]

**Class Examples:**

Divide the following.

\[
\begin{array}{c|c}
\frac{3}{5} & \div & \frac{2}{3} \\
\hline
\frac{2}{7} & \div & \frac{1}{4}
\end{array}
\]
Try on your own:
Divide the following.

<table>
<thead>
<tr>
<th>( \frac{5}{6} \div \frac{10}{12} )</th>
<th>( \frac{2}{4} \div 7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{2} \div \frac{1}{5} )</td>
<td>( \frac{4}{3} \div \frac{5}{5} )</td>
</tr>
</tbody>
</table>

Applications:

Mr. Stewart has 28 licorice sticks. Each student in his class receives \( \frac{1}{6} \) of a licorice stick.
How many students are in his class?
Multiplying & Dividing Fractions

1. Find the product. Show your work.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$\frac{3}{4} \times \frac{1}{3}$</td>
<td>b.</td>
</tr>
<tr>
<td>c.</td>
<td>$\frac{5}{9} \times 6\frac{1}{4} \times \frac{1}{3}$</td>
<td>d.</td>
</tr>
<tr>
<td>e.</td>
<td>$4\frac{3}{4} \times 2\frac{1}{3}$</td>
<td>f.</td>
</tr>
<tr>
<td>g.</td>
<td>$2\frac{2}{5} \times 1\frac{1}{4} \times 2$</td>
<td>h.</td>
</tr>
</tbody>
</table>
2. Find the quotient. Show your work.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a. | \[
\frac{3\frac{2}{3}}{4} \div \frac{5}{3}
\]
| b. | \[
\frac{3}{4} \div 5\frac{1}{2}
\]
| c. | \[
6 \div \frac{4}{9}
\]
| d. | \[
\frac{3}{4} \div 12
\]
| e. | \[
\frac{2}{5} \div \frac{4}{7}
\]
| f. | \[
\frac{3}{4} \div 5\frac{1}{2}
\]
3. Three friends share a chocolate bar. The first friend takes $\frac{1}{2}$ of the bar and the second takes $\frac{1}{3}$ of the amount that is left. How much of the original bar will the third person receive?

4. Trent feeds his cattle $\frac{2}{7}$ of a square bale each. He has 84 cattle. How many bales will he need to feed his cattle?

5. A recipe calls for $2\frac{1}{2}$ cups of chocolate chips to make a batch of five dozen cookies. On average, how many cups of chocolate chips would be in each cookie? What assumption are we making?
1. You are going to the corner store. The following items can be purchased:
   (Remember you are at the store without a calculator… please use your estimation skills).

   Small bag of chips $1.29
   Chocolate Bar . $0.89
   Package of Licorice $3.43
   Chocolate Milk (500 mL) $1.15
   White Milk (500 mL) $1.15
   Bottle of Water $2.41

   a) How much would it cost to buy a chocolate bar and a bottle of water?
      Show the estimation that you used to arrive at your answer.

   b) If you bought 3 chocolate milks, 3 bags of chips and a pack of licorice. Would $10 be enough?

2. Compare the following decimals, circle the largest value.

   a) 0.8 or 0.65
   b) 14.5 or 14.52
   c) -1.61 or -1.68

3. Complete the following operations:

   a) 34.51 + 5.39 =
   b) 524.79 − 32.85 =
   c) 9 − 3.25 =
d) \[ 780.05 + 17.9 = \] 

e) \[ 2.1 \times 3 = \] 

f) \[ 8.5 \times 4.7 = \]

4. Manuel went to the store and bought three pairs of jeans that cost $29.89 each. What is the total cost of his purchase?

5. Ishmael’s dad purchases gas for his car 5 times each month. Gas costs 97.7 cents per litre and he purchases 65 litres each time. How much does he spend on gas each month?

6. | Fraction out of 100 | Decimal | Percent |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{4} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{4}{5} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{12}{50} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. The flag shown below was designed with four different patterns. Determine the fraction of the flag (in lowest terms) that is:

   a) Orange
   b) Checkered
   c) Solid
   d) Striped
8. \(\frac{20}{32}\) is an equivalent fraction for \(\frac{5}{8}\). Write two more equivalent fractions to \(\frac{5}{8}\).

9. Circle the fractions that sit between \(\frac{7}{16}\) and \(\frac{9}{16}\).

\[
\begin{array}{cccccccc}
\frac{13}{32} & 1 & \frac{3}{4} & \frac{1}{2} & \frac{5}{8} & \frac{3}{8} & \frac{19}{32}
\end{array}
\]

10. Complete the following operations:

   a) \(\frac{1}{6} + \frac{1}{3} = \)  
   b) \(\frac{5}{6} - \frac{1}{4} = \)  
   c) \(1\frac{1}{3} + 2\frac{3}{4} = \)

   d) \(\frac{3}{10} \times \frac{1}{6} = \)  
   e) \(\frac{8}{3} + \frac{24}{20} = \)  
   f) \(3\frac{1}{2} \times \frac{5}{21} = \)

11. Chloe collected 4 times as many bags of cans as her friend. If her friend collected \(\frac{1}{6}\) of a bag, how much did Chloe collect?

12. Adam had a lump of silly putty that was \(\frac{4}{5} \frac{1}{6}\) inches long. If he stretched it out to \(2\frac{2}{3}\) times its current length how long would it be?
Unit 2: Coordinates & Equations
The Coordinate System

Definitions:

The **x-axis** is the horizontal line or axis on a graph. It represents the left and right movement.

The **y-axis** is the vertical line or axis on a graph. It represents the up and down movement.

The **origin** is point at which the x axis and y axis intersect.

**Coordinates** or **ordered pairs** are pairs of numbers that describe the location of a point on the coordinate plane compared to the origin.

The **x-coordinate** is the first number in the ordered pair. It describes the distance along the x-axis from the origin.

The **y-coordinate** is the second number in the ordered pair. It describes the distance along the y-axis from the origin.

Plotting coordinates (x, y)

- The first number tells us how many places from the origin to move left (-) or right (+).
- The second number tells us how many places to then move down (-) or up (+).
- Example, to plot (-3, 9) you start at the origin and move 3 units left & then 9 units up.
- Example, to plot (5, -7), you start at the origin and move 5 units right & then 7 units down.
Quadrants

The x and y axis on the coordinate grid create four identical regions.

These regions are referred to as Quadrant I, II, III & IV.

Class Example:

a) Plot and label the points.
   A (-3, 7)    B (5, 6)
   C (-1, -5)  D (8, -9)

b) Name the coordinates of the points below:
   P (___, ___)  Q (___, ___)
   R (___, ___)  S (___, ___)

Try on your own:

a) Plot and label the points.
   E(-4, 6)    F(0, 8)
   G(-4, -8)  H(4, 0)

b) Name the quadrant or axis that each point is in/on:
   E          F
   G          H
## Plotting points from a table of values

- The number in the first column represents \( x \)-values
- The number in the second column represents \( y \)-values

### Class Example:

**a)** Plot and label the points.

<table>
<thead>
<tr>
<th></th>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-5</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

**b)** Name the quadrant or axis each point lies in/on:

- A
- B
- C
- D

### Try on your own:

**a)** Plot and label the points.

<table>
<thead>
<tr>
<th></th>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>-2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**b)** Name the quadrant or axis each point lies in/on:

- E
- F
- G
- H
Graph the following points in the order outlined below reading left to right. When you plot one coordinate, connect it to the next coordinate with a straight line.

<table>
<thead>
<tr>
<th>Start</th>
<th>(3, -21)</th>
<th>(1, -16)</th>
<th>(7, -13)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-8, -10)</td>
<td>(4, -11)</td>
<td>(1, -16)</td>
</tr>
<tr>
<td></td>
<td>(-1, -11)</td>
<td>(-6, -8)</td>
<td>(-7, -5)</td>
</tr>
<tr>
<td></td>
<td>(-4, -6)</td>
<td>(-1, -11)</td>
<td>(4, 4)</td>
</tr>
<tr>
<td></td>
<td>(4, -3)</td>
<td>(7, -5)</td>
<td>(8, -2)</td>
</tr>
<tr>
<td></td>
<td>(4, 4)</td>
<td>(11, 0)</td>
<td>(14, 2)</td>
</tr>
<tr>
<td></td>
<td>(12, 4)</td>
<td>(4, 4)</td>
<td>(10, 6)</td>
</tr>
<tr>
<td></td>
<td>(11, 8)</td>
<td>(9, 9)</td>
<td>(4, 4)</td>
</tr>
<tr>
<td></td>
<td>(6, 9)</td>
<td>(5, 11)</td>
<td>(3, 9)</td>
</tr>
<tr>
<td></td>
<td>(4, 4)</td>
<td>(0, 10)</td>
<td>(-5, 10)</td>
</tr>
<tr>
<td></td>
<td>(-3, 7)</td>
<td>(4, 4)</td>
<td>(-2, 3)</td>
</tr>
<tr>
<td></td>
<td>(-3, 0)</td>
<td>(0, 0)</td>
<td>(4, 4)</td>
</tr>
</tbody>
</table>
2. Graph each section as though it were a separate picture but on the same graph paper. Connect the dots in the order that you graph them moving from left to right.

**Section One**

- (3, 0)
- (-8, 1)
- (-9, -9)
- (1, -11)
- (3, 0)

- (1, 2)
- (-9, 0)
- (-8, -10)
- (2, -10)

- (-1, 3)
- (-10, -2)
- (-7, -11)
- (3, -9)

- (-3, 3)
- (-11, -4)
- (-5, -12)
- (4, -8)

- (-5, 3)
- (-11, -6)
- (-3, -12)
- (5, -6)

- (-7, 2)
- (-10, -8)
- (-1, -12)
- (5, -2)

**Section Two**

- (1, 2)
- (-1, 9)
- (-3, 17)
- (3, 17)

- (2, 2)
- (-2, 10)
- (-3, 15)
- (2, 18)

- (3, 3)
- (-4, 12)
- (-2, 14)
- (1, 19)

- (3, 5)
- (-6, 14)
- (0, 13)
- (-1, 19)

- (2, 7)
- (-6, 16)
- (2, 14)
- (-2, 18)

- (1, 8)
- (-5, 17)
- (3, 15)
- (-3, 17)

**Section Three**

- (3, 17)
- (-11, 0)
- (-3, -12)
- (-3, -12)

- (5, 15)
- (-9, 2)
- (-3, -14)
- (-5, -22)

- (6, 13)
- (-11, 4)
- (-3, -16)
- (-1, -22)

- (5, 11)
- (-13, 5)
- (-3, -18)
- (-3, -20)

- (4, 13)
- (-15, 5)
- (-3, -20)
- (-5, -22)

- (3, 14)
- (-14, -3)
- (-12, -2)
- (-3, -20)

Shade in the resulting shape

**Section Four**

- (1, 17)
- (-11, 0)
- (-3, -12)
- (-3, -12)

- (2, 17)
- (-13, 0)
- (-5, -14)
- (-5, -14)

- (2, 16)
- (-15, -1)
- (-7, -16)
- (-5, -16)

- (1, 16)
- (-16, -3)
- (-5, -17)
- (-3, -17)

- (1, 17)
- (-17, -5)
- (-3, -17)
- (-1, -17)

- Shade in the resulting shape

**Section Five**

- (-8, 1)
- (-13, 2)
- (-11, 0)
- (-14, -3)

- (-9, 2)
- (-11, 0)
- (-9, 0)
- (-12, -2)

- (-11, 4)
- (-9, 0)
- Shade in the resulting shape.

**Section Six**

- (-11, 0)
- (-13, 0)
- (-15, -1)
- (-15, -4)

- (-13, 0)
- (-11, 0)
- (-9, 0)
- (-10, -2)

- (-15, -1)
- (-16, -3)
- (-17, -5)
- Shade in the resulting shape

- (-15, -4)
- (-17, -5)
- (-15, -4)
- (-16, -3)

**Section Seven**

- (-3, -12)
- (-1, -22)
- (-3, -12)
- (-3, -14)

- (-3, -16)
- (-3, -16)
- (-3, -20)
- (-3, -20)

- (-3, -18)
- (-3, -18)
- (-5, -22)
- (-5, -22)

- (-3, -20)
- (-5, -22)
- (-5, -22)
- (-5, -22)

Shade in the resulting shape

**Section Eight**

- (-3, -12)
- (-1, -15)
- (-3, -12)
- (-5, -14)

- (-5, -14)
- (-7, -16)
- (-1, -17)
- (-5, -17)

- (-7, -16)
- (-5, -17)
- (-3, -17)
- (-1, -17)

- (-5, -17)
- (-3, -17)
- (-1, -17)
- Shade in the resulting shape
3. Make your own design with a minimum of 20 points on the graph paper below. Beneath it create a table of values and instructions to replicate your design.

Trade your table of values and instructions with a partner and graph each other’s design.

Instructions:
Table of Values

Concepts:
- Recognize a pattern, in a given table of values, to help complete missing information in the table.
- Substitute values into a given formula to evaluate the expression or equation.

Self-Check:
- □ I can identify a pattern in a table of values and use it to finish the table
- □ I can substitute into a given formula to evaluate an expression or equation

Completing a Table of Values:

When given a table of values that is incomplete we must solve for the missing numbers before we can convert to ordered pairs and plots the points on the Cartesian Plane.

**Method 1: Identifying & Using a Pattern:**

If no equation is given, analyze the table of values to find a pattern in the numbers.

**Steps:**
1. Fill in the missing values in the tables, by recognizing the pattern.
2. Convert the table of values to ordered pairs.
3. Plot the points.

**Class Example:**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

**Try on your own:**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>
**Using the given Equation:**
Often an equation will also be given with the table of values. If this is the case we can use this equation to help fill in the table of values. The process used is called **Substitution**.

**Substitution** means to replace a variable with a given value and use order of operations to evaluate the expression.

**Substitution Class Examples:**
Evaluate the following.

| 7x + 3y for x = 2 and y = -3 | \( \frac{-d}{2r} \) for d = 8 and r = 2 |

**Substitution Try on your own:**
Evaluate the following.

| -5x² + 12y - 2 for x = 3 and y = -1 | -3(h + b) for h = -5 and b = 2 |

**Method 2: Using the Equation**

Steps:  
1. Enter a known value from the table in to the equation.  
   (This will result in an equation with only one unknown)  
2. Using your order of operations evaluate and solve for the unknown variable.  
3. Place this value in the table and repeat for all missing values.

**Equation Method Class Examples:**
Evaluate the following.

| y = x - 9 | y = 3x + 1 |
| x = 2 y = ____ | x = -4 y = ____ |

| x = 10 y = ____ | x = 0 y = ____ |
### Class Example:
Complete the table of values to write the ordered pairs for the given relation.

\[ y = 2x - 4 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

### Try on your Own:
Complete the table of values to write the ordered pairs for the given relation.

\[ y = x + 3 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

### Try on your own:
Complete the table of values to write the ordered pairs for the given relation.

\[ x + y = 3 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>-3</td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>
### Table of Values

1. Evaluate the following for the indicated values.

   a) \(3 - 4x\) for \(x = -2\)

   b) \(-x + 5y\) for \(x = -3\) and \(y = -6\)

   c) \(-2ab - 5ab\) for \(a = 5\) and \(b = -3\)

   d) \(x^2 + 7gk\) for \(x = -2\), \(g = 9\) and \(k = -1\)

2. Use the given equation to complete the table of values.

   a. \(y = x - 5\)

   b. \(x + y = -4\)

   c. \(x - y = 2\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td>-2</td>
<td></td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>-1</td>
<td></td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

3. Use the relation \(2x + y = 10\) to find the missing values.

   a. \((4, ??)\)

   b. \((-2, ??)\)

   c. \((??, 5)\)

   d. \((??, -4)\)
Graphing a Relation from an Equation

**Steps:**
1. Choose 5 values for $x$ (ideally 2 positive, 0, and 2 negative).
2. Substitute each value of $x$ in the equation and find the matching $y$-value.
3. Pair the $x$ and $y$ values together to create 5 ordered pairs.
4. Graph the coordinates using a suitable scale.

**Example:** Graph 5 ordered pairs for the relation $y = x - 3$

**Step One:** Choose 5 values for $x$ (2 positive, 0, and 2 negative).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Step Two:** Substitute each value of $x$ in the equation and find the matching $y$-value.

**Step Three:** Pair the $x$ and $y$ values together to create 5 ordered pairs.

(-2, -5)  (-1, -4)  (0, -3)  (1, -2)  (2, -1)

**Step Four:** Graph the coordinates using a suitable scale and connect the dots.
Class Examples:
Graph 5 ordered pairs that satisfy each relation.

\[ y = 1 - x \]

\[ y = 2x + 3 \]
Try on your own:
Graph 5 ordered pairs that satisfy each relation.

\[ y = 2x - 1 \]

\[ x - y = -2 \]
Finding an Equation from a Table of Values

Steps: 1. Find a pattern that can be applied between the columns.  
* In other words, find a rule that will always connect values in the first column to values in the second column.

2. Write the equation for one variable in terms of the other.  
* Most commonly takes the form $y = \ldots$

3. Check to see if the equation works for the entire table.

Example: Write an equation for the relation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

Step One: Find a pattern that can be applied between the columns.

* How can you move from:

1 —— 3  
2 —— 5  
3 —— 7  
4 —— 9

Step Two: Write the equation for one variable in terms of the other.

$$y = 2x + 1$$

Step Three: Check to see if the equation works for the entire table.

Check: (1,3)  
$y = 2x + 1$  
$3 = 2(1) + 1$  
$3 = 3$

Check: (2,5)  
$y = 2x + 1$  
$5 = 2(2) + 1$  
$5 = 5$

Check: (3,7)…
**Class Example:**
Write an equation for the relation & verify your equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

**Try on your own:**
Write an equation for each relation & verify your equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>
1. Determine the algebraic equation that describes the relationship between $x$ and $y$ in the following table of values.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>-2</td>
<td>14</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-7</td>
</tr>
<tr>
<td>2</td>
<td>-14</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

2. Complete the table of values and graph each relation.

(a) $y = -2x + 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(b) $x + y = 4$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. \( y = -\frac{2}{3}x + 4 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
Unit 2: Coordinates & Equations  Lesson 15

Slope

VIDEO: Math Mash-Up: What is Slope?
https://tinyurl.com/zbvf2h7
(6:49 minutes in length)

This video will introduce you to all of the concepts that will be covered today.

Including:

• The meaning of slope
• Line steepness and direction
• How do I graph a line?
• How do I find the slope of a line?
• How do I use the slope formula?
• What is the difference between a positive slope and a negative slope?
• How do I simplify slope?

Concepts:
• Introduction to slope pictorially, conceptually and algebraically.
• Four types of slope: positive, negative, zero & undefined – are we addressing these now?.
• Introduction and application of slope formula.
• Interactions between slope and table of values.
• Slope and graphs of lines.

Self-Check:
☐ I can recognize the slope of a line given the graph, pattern, or table of values
☐ I can recognize if a slope is positive, negative, zero or undefined
☐ I can apply the slope formula correctly when given two coordinates
☐ I can recognize how changing slope will impact the table of values
☐ I can create a graph of a line given the slope
Finding the slope of a line & Graphing the line:

Steps:

1. Identify two points on the line and write the ordered pairs of the two points.

2. Label the first point \((x_1, y_1)\) and the second point \((x_2, y_2)\)

3. Substitute the two ordered pairs in to the slop formula

\[
\text{Slope Formula: } \frac{\Delta \text{ rise}}{\Delta \text{ run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}
\]

4. Starting from one of the given points, use the slope (rise/run) to graph the line.

If you're given two points
\((x_1, y_1)\) and \((x_2, y_2)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

NOTICE: The two coordinates should be stacked one on top of the other. \(y_2 \) & \(x_2\) together and \(y_1 \) & \(x_1\) together.

Class Example:
Determine the slope a line passing through \((1, 3)\) and \((7, 12)\).

Step 1: Identify two points on the line:

Point 1: \((1, 3)\)
Point 2: \((7, 12)\)

Step 2: Label the points:

\((1, 3)\) \quad \((7, 12)\)
\(x_1, y_1\) \quad \(x_2, y_2\)

Step 3: Use slope formula:

\[
\text{Slope} = \frac{(12 - 3)}{(7 - 1)} = \frac{9}{6} = \frac{3}{2}
\]

Step 4: Start from one point and use the slope to graph the line.

Plot \((1, 3)\) as a starting point.

Use the slope to graph the line; go up 3 and right 2 to get to the next points.
**Try on your own:**
Find the slope of the following lines, then plot them onto the same graph, using different colours. There should be at least four points for each line clearly shown.

<table>
<thead>
<tr>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,4) and (9,7)</td>
<td>(0,3) and (1,0)</td>
</tr>
<tr>
<td>(-9, -3) and (0,0)</td>
<td>(-7, +9) and (+2, -9)</td>
</tr>
</tbody>
</table>
Special Cases for Slope:

<table>
<thead>
<tr>
<th></th>
<th>When a line segment increases to the right, both the x and y coordinates will increase. Both the rise and the run are positive so the line segment has a positive slope.</th>
<th>When a line segment decreases to the right, the x coordinate increases, while the y coordinate decreases. Since the x and y coordinates are moving in opposite directions, the line segment has a negative slope.</th>
<th>When a line segment moves horizontally to the right, without either increasing or decreasing vertically, the line segment has a zero slope.</th>
<th>When a line segment moves vertically up and down, without either increasing or decreasing horizontally, the slope of the line segment is undefined.</th>
</tr>
</thead>
</table>

For Example:

\[
\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(3 - [-3])}{(3 - [-1])}
\]

\[
= \frac{6}{4} = \frac{3}{2}
\]

For Example:

\[
\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{([-3] - 3)}{(3 - [-3])}
\]

\[
= \frac{0}{6} = 0
\]

For Example:

\[
\frac{([-1] - [-1])}{(3 - [-3])}
\]

\[
= \frac{0}{6} = 0
\]

Try on your own:

Find the slope of the following lines:

\((-6,7) \text{ and } (4,7)\)

\((1,-3) \text{ and } (1,5)\)
Determining the Slope from a graph:

Steps:
1. Identify (minimum of 2) points on your graph. It is best if these points pass through exact values (i.e. 2 & -5… not estimating)
2. Count the rise & run between these two (or more) points.
3. State the slope in lowest form.
4. Ensure the sign of the slope matches the line, using what you know about + and – lines.

Class Example:
Determine the slope of the following lines:

Try on your Own:
Determine the slope of the following lines:
1. You know that a line goes through the point \((4, 2)\) and that it slants up and to the right. Name at least one other thing that you are sure is TRUE about the line. What could the slope of this line be?

2. Find the slope of the following lines, using the slope formula.

<table>
<thead>
<tr>
<th>((3, -20)) and ((5, 8))</th>
<th>((1, -19)) and ((-2, -7))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((20, 8)) and ((9, 16))</td>
<td>((19, 3)) and ((20, 3))</td>
</tr>
<tr>
<td>((6, -12)) and ((15, -3))</td>
<td>((17, -13)) and ((17, 8))</td>
</tr>
</tbody>
</table>
3. Match each graph to the correct slope. (Write the slope on/beside the graph it goes with).

A. \( \frac{1}{2} \)

B. \( \frac{-3}{2} \)

C. \( \frac{-5}{3} \)

D. \( \frac{-5}{4} \)

E. 0

F. \( -1 \)

G. 4

H. undefined
Unit 2: Coordinates & Equations

Lesson 16

Solving Linear Equations

Concepts:
- Model and solve, concretely, pictorially and symbolically, one step linear equations.
- Model and solve, concretely, pictorially and symbolically, two step linear equations.

Self-Check:
- I can solve one step equations that involve addition & subtraction
- I can solve one step equations that involve multiplication & division
- I can solve two step problems that involve a combination of operations

Definitions:

*Algebraic equation* is an equation that contains variables.

For example, \( x + 2 = 4 \) is an algebraic equation.

*Solving* an algebraic equation means to find the correct value of the variable that makes the equation “true” or balanced.

For example, the solution to the equation \( n + 5 = 8 \) is \( n = 3 \) because \( 3 + 5 = 8 \).

An *inverse operation* is an operation that “undoes” another operation. It is its opposite.

For example, the inverse operation of *subtraction* is *addition*.

the inverse operation of multiplication is ____________.

*Checking* a solution to an algebraic equation means to substitute the value of the variable in the original equation and use order of operations to see if you reach a true statement.

For example, \( 2x - 3 = 7 \) you determine has a solution of \( x = 5 \)

Check your solution by substituting 5 in for \( x \).

\[
\begin{align*}
2x & - 3 = 7 \\
2(5) & - 3 = 7 \\
10 & - 3 = 7 \\
7 & = 7 \\
\text{Check complete!}
\end{align*}
\]
Solving One-Step Equations

Operation: Addition/Subtraction

Reminder:
Addition and Subtraction are inverse operations.
Whatever you do to one side you must also do to the other!

Class Examples:
Solve the following:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 5 = 2 )</td>
<td>( -m - 3 = 3 )</td>
</tr>
</tbody>
</table>

Try on your Own:
Solve the following:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w - 7 = -3 )</td>
<td>( -x + 24 = -1 )</td>
</tr>
</tbody>
</table>

Operation: Multiplication/Division

Steps:
1. Identify the operation (O) and the inverse operation (I).
2. Perform the inverse operation to both sides of the equation.
3. Solve the equation and state your final answer.
4. Check the solution in the original equation.
**Multiplication Example:** \(5y = 35\)

**Step One:** Identify the operation (O). Identify the inverse operation (I).

\[
\begin{align*}
5y &= 7 \quad \text{O: } \times 5 \\
5y &= 7 \quad \text{I: } \div 5
\end{align*}
\]

Recall: ‘5y’ means five multiplied by variable y.

**Step Two:** Perform the inverse operation to both sides of the equation.

\[
\frac{5y}{5} = \frac{35}{5}
\]

Note: \(5y \div 5 = 1y\) divide the numbers

**Step Three:** Solve the equation and state your final answer.

\(y = 7\)

**Step Four:** Check the solution in the original equation.

\[
\begin{align*}
5y &= 35 \\
5(7) &= 35 \\
35 &= 35
\end{align*}
\]

**Class Examples:**

Solve the following algebraically. Show all work and a proper check.

<table>
<thead>
<tr>
<th>(9x = -27)</th>
<th>(-4x = 12)</th>
</tr>
</thead>
</table>

| \(\frac{1}{4}x = 12\) | \(-5x = -15\) |
Try on your own:
Solve the following algebraically. Show all work and a proper check.

<table>
<thead>
<tr>
<th>6x = 54</th>
<th>-4x = 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3x = 1/9</td>
<td>2x = -12</td>
</tr>
</tbody>
</table>

**Division Example:** \( \frac{x}{6} = 3 \)

**Step One:** Identify the operation (O). Identify the inverse operation (I).

\[
\frac{x}{6} = 3 \quad \text{O: ÷6} \\
\frac{x}{6} = 3 \quad \text{I: × 6}
\]

**Step Two:** Perform the inverse operation to both sides of the equation.

\[
6\left(\frac{x}{6}\right) = 6(3) \quad \text{Note: } 6\left(\frac{x}{6}\right) = 1x \text{ because the 6’s cancel } \left(\frac{x}{6}\right)
\]

**Step Three:** Solve the equation and state your final answer.

\[x = 18\]

**Step Four:** Check the solution in the original equation.

\[
\frac{x}{6} = 3 \\
\frac{(18)}{6} = 3 \\
3 = 3 \sqrt{3}
\]
### Class Examples:
Solve the following algebraically. Show all work and a proper check.

<table>
<thead>
<tr>
<th>( \frac{x}{6} = 7 )</th>
<th>( \frac{x}{4} = -9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{3} = 4 )</td>
<td>( \frac{x}{5} = -1 )</td>
</tr>
</tbody>
</table>

### Try on your own:
Solve the following algebraically. Show all work and a proper check.

<table>
<thead>
<tr>
<th>( \frac{x}{-2} = -84 )</th>
<th>( \frac{x}{-3} = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{5} = 10 )</td>
<td>( \frac{x}{4} = 6 )</td>
</tr>
</tbody>
</table>
### Solving One Step Equations
#### Assignment

1. Solve and check the following:

<table>
<thead>
<tr>
<th>a. ( x - 12 = 8 )</th>
<th>b. ( b + 5 = -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 20 )</td>
<td>( b = -8 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. ( n + 3 = 9 )</th>
<th>d. ( -x - 6 = -6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 6 )</td>
<td>( x = 0 )</td>
</tr>
</tbody>
</table>

2. Solve and check the equation. Show all of your work.

<table>
<thead>
<tr>
<th>a. ( x - 12 = -15 )</th>
<th>b. ( -3 = b + 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -3 )</td>
<td>( b = -15 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. ( -n + 9 = 7 )</th>
<th>d. ( x - 5 = 11 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 1 )</td>
<td>( x = 16 )</td>
</tr>
</tbody>
</table>
3. Solve and check the equation. Show all of your work.

<table>
<thead>
<tr>
<th>a. $\frac{x}{2} = -10$</th>
<th>g. $4b = -12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. $\frac{x}{5} = -20$</td>
<td>h. $5x = -15$</td>
</tr>
<tr>
<td>c. $\frac{x}{6} = 12$</td>
<td>i. $11b = -77$</td>
</tr>
<tr>
<td>d. $\frac{x}{7} = -35$</td>
<td>j. $4x = 84$</td>
</tr>
<tr>
<td>e. $\frac{m}{-4} = -24$</td>
<td>k. $\frac{x}{2} = 4$</td>
</tr>
<tr>
<td>f. $-5 = \frac{n}{15}$</td>
<td>l. $-2 = \frac{n}{20}$</td>
</tr>
</tbody>
</table>
Solving Two (or more) Step Equations:

Steps:  
1. Use operations and inverse operations to solve the equation starting with the operations furthest from the variable.

2. State your final answer.

3. Check the solution in the original equation.

Example: \( \frac{7x}{3} - 5 = 30 \)

Step One: Use operations and inverse operations to solve the equation starting with the operations furthest from the variable.

\[
\begin{align*}
\frac{7x}{3} - 5 & = 30 & \text{O: } -5 \\
+5 & +5 & \text{I: } +5 \\
\frac{7x}{3} & = 35 & \text{O: } \div 3 \\
3\left(\frac{7x}{3}\right) & = 3 \times 35 & \text{I: } \times 3 \\
7x & = 105 & \text{O: } \times 7 \\
\frac{7x}{7} & = \frac{105}{7} & \text{I: } \div 7
\end{align*}
\]

Step Two: State your final answer.

\( x = 15 \)

Step Three: Check the solution in the original equation.

\[
\begin{align*}
\frac{7x}{3} - 5 & = 30 \\
\frac{7(15)}{3} - 5 & = 30 \\
105 \div 3 - 5 & = 30 \\
35 - 5 & = 30 \\
30 & = 30 \checkmark
\end{align*}
\]
### Class Examples:
Solve and check.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 9a - 6 = 21 )</td>
<td>( \frac{x}{2} + 5 = 7 )</td>
</tr>
<tr>
<td>( 12 = \frac{x}{8} + 5 )</td>
<td>( -4.2x + 4 = -8 )</td>
</tr>
</tbody>
</table>

### Try on your own:
Solve and check.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8x + 7 = 15 )</td>
<td>( 7 + \frac{x}{3} = 9 )</td>
</tr>
<tr>
<td>( -18x + 5 = 13.2 )</td>
<td>( 27 = 5 + 2x )</td>
</tr>
</tbody>
</table>
Solving Equations with Variables on Both Sides

Steps: 1. Group like terms together on the same side.
   * Variables on one side & constants on the other.
   ** It can also be helpful to keep the variables on the side in which they are positive.

2. Combine your like terms.

3. Use inverse operations to solve the equation, starting with operations furthest from the variable.

4. State your final answer.

5. Check the solution in the original equation.

Example: $8x - 10 = -2x - 20$

Step One: Group like terms together on the same side.

Step Two: Combine your like terms.

Step Three: Solve the equation.

Step Four: Check the solution in the original equation.
Class Examples:
Solve and check.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4c + 2 = 3c + 5$</td>
<td>$6x + 4x = 16 + 2x$</td>
</tr>
</tbody>
</table>

Try on your own:
Solve and check.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x - 7 = 2x + 3$</td>
<td>$2x - 6x + 8 = -4 - 2x$</td>
</tr>
</tbody>
</table>
Applications:

When presented with a word problem, our **first step** is to convert the words to the equations. Once this is done we can move to the **second step:** solving the equation.

Here are some key words to look for when converting WORDS to EQUATIONS:
Steps for Solving Word Problems:

1. Identify the variable, the quantity that is unknown, and properly define it.
2. Identify the words that lead to a mathematical operation.
3. Write the equation.
4. Solve and check.
5. Go back and answer the question.

Example:
Five added to three times a number equals one hundred and thirteen. Find the number.

Step One: Identify the variable, the quantity that is unknown, and define it.

\[
\text{a number} \quad x
\]

Step Two: Identify the words that lead to a mathematical operation.

ADDED +
TIMES ×
EQUALS =

Step Three: Write the equation.

\[ 5 + 3x = 113 \]

Step Three: Solve and check.

\[
\begin{align*}
5 + 3x &= 113 \\
-5 &\quad -5 \\
3x &= 108 \\
\div 3 &\quad \div 3 \\
x &= 36
\end{align*}
\]

Check:
\[
\begin{align*}
5 + 3(36) &= 113 \\
5 + 108 &= 113 \\
113 &= 113 \checkmark
\end{align*}
\]

Step Four: Go back and answer the question.

The number is 36.
Class Example:
Solve the word problem using all five steps listed.

12 is added to four times a number which results in a total of fifty two. Find the number.

Try on your own:
Solve the word problem using all five steps listed.

One number is five times as large as another. Their sum is seventy eight. What is the smaller number?
Example:
The perimeter of a rectangle is 234 m. The rectangle is twice as long as it is wide. What is its length and its width?

Step One: Identify the variables. (Draw a sketch and record the information on the diagram.)

width in m = \(x\)
length in m = 2\(x\)

Step Two: Write the equation.

\[2x + 2(2x) = 234\]

Step Three: Solve and check.

\[
\begin{align*}
2x + 2(2x) &= 234 \\
2x + 4x &= 234 \\
6x &= 234 & \text{Expand.}
\end{align*}
\]

\[
\begin{align*}
6x &= 234 & \text{Group like terms.}
\end{align*}
\]

\[
\begin{align*}
&\frac{6x}{6} = \frac{234}{6} & \text{O: } \times 3 \quad \text{I: } \div 3
\end{align*}
\]

\[x = 39\]

Check:

\[
\begin{align*}
2(39) + 2[2(39)] &= 234 \\
78 + 156 &= 234 \\
234 &= 234 \quad \checkmark
\end{align*}
\]

Step Four: Go back and answer the question.
The width is 39 meters \((x)\) and the length is 78 meters \((2x)\).

Class Example:
Solve the word problem using the five steps listed.

The perimeter of a rectangle is 89 cm. The length is 8 cm more than the width. What is the width?
Try on your own:
Solve using the five steps listed above.

A rectangle, whose perimeter is one hundred seventy-six cm, has a length that is six cm longer than its width. What are the dimensions of the rectangle?

Example:
If the perimeter of the following triangle is 294 m, find the measure of each of its sides.

```
Step One: Identify the variables. (Draw a sketch and record the information on the diagram.)
Diagram is already labeled.

Step Two: Write the equation.
\[ x + 5 + 6x + 2x + 10 = 294 \]

Step Three: Solve and check.
\[
\begin{align*}
x + 5 + 6x + 2x + 10 &= 294 \\
9x + 15 &= 294 \\
9x &= 279 \\
x &= 31
\end{align*}
\]

Check:
\[
\begin{align*}
(31) + 5 + 6(31) + 2(31) + 10 &= 294 \\
(31) + 5 + 186 + 2(31) + 10 &= 294 \\
(31) + 5 + 186 + 62 + 10 &= 294 \\
36 + 186 + 62 + 10 &= 294 \\
294 &= 294 \checkmark
\end{align*}
\]

Step Four: Go back and answer the question.
The first side is 36 m \((x + 5)\), the second side is 186 m \((6x)\) and the third side is 72 m \((2x + 10)\).
Class Example:
Solve using the five steps listed.

Determine the value of $x$ given:

![Diagram](image1)

$P = 143 \text{ cm}$

Class Example:
Solve using the five steps listed.

Determine the value of $x$ given:

![Diagram](image2)

$Area = 15$
Use the 5-step process listed in this lesson to solve each word problem.

1. Identify and properly define the variable(s)
2. Write an equation to represent the situation
3. Solve the equation
4. Check your answer
5. State the solution by answering the initial question.

1. Seven added to three times the number is equal to forty-three. Find the number.

2. Three times a number decreased by eight is equal to the same number decreased by twenty two. Find the number.

3. If forty is added to a number, the sum is half of the product of the number and twelve. Find the number.

4. Find the number which when divided by four and increased by twelve is the same as when it is decreased by five.
5. A rectangle with a perimeter of 32 cm has a length that is 1 cm more than twice its width. Find the dimensions.

\[ P = 32\text{cm} \]

6. The square and the triangle have the same perimeter in centimeters. What are their dimensions?

\[
\begin{align*}
triangle & \quad k + 2 \\square & \quad 2k - 1
\end{align*}
\]

For each of the following, find the length of each side when the perimeter of the figure is 48 cm.

7. \[
\begin{align*}
\text{rectangle} & \quad x + 5 \\
x + 4 & \quad 2x
\end{align*}
\]

11. \[
\begin{align*}
\text{hexagon} & \quad n + 5
\end{align*}
\]
This review will cover:
Lesson 12: Coordinate System & Ordered Pairs
Lesson 13: Table of Values
Lesson 14: Equations
Lesson 15: Slope
Lesson 16: Solving Linear Equations

1. Using the following equations:
   i. Complete a table of values including at least 5-points.
   ii. Graph the points on the grids provided.
   a) \( y = 2x - 3 \)  
   b) \( x - 2y = 8 \)

2. Create an algebraic expression that represents the relationship between \( x \) and \( y \) in the following table of values.
   a) \[
   \begin{array}{c|c}
   X & Y \\
   \hline
   2 & 1 \\
   4 & 2 \\
   6 & 3 \\
   8 & 4 \\
   \end{array}
   \]
   b) \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 3 \\
   1 & 8 \\
   2 & 13 \\
   3 & 18 \\
   4 & 23 \\
   \end{array}
   \]
3. Determine the slope:

<table>
<thead>
<tr>
<th>Question</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between points (2,2) and (6,4)</td>
<td></td>
</tr>
</tbody>
</table>

4. Solve the following linear equations.

a) $x + 7 = -6$

b) $x + 2 = -2$

c) $-x + 8 = 9$

d) $9 - x = 3$

e) $10x + 8 = 7$

f) $5x - 1 = 9$

g) $9 - x + 5x = -3$

h) $2x + 4 = -7x - 4$

i) $3(x + 2) = 12$
5. Tom has a comic book collection. He sold half of his collection then bought 6 more. He now has 12 comic books, how many did he originally have?

6. When four times a number is subtracted from 16, the result is 32. What is the number?

7. If you double a number and then add 36 you get five times the number. What is the original number?

8. Andy is baking a rectangular cake with a perimeter of 76cm. The length of the cake is four less than double the width. What are the dimensions of the cake?
Unit 3: Polynomials
Unit 4: Polynomials

Introduction to Polynomials: Part I

Concepts:
- Identify the different parts of a polynomial expression; constant, coefficient, variable.
- Classify polynomials by number of terms and/or degree.
- Identify and combine the like terms in a polynomial expression.

Self-Check:
- I can identify all parts of a polynomial expression including; constant, coefficient & variables.
- I can classify and name a polynomial by the number of terms.
- I can classify a polynomial by its degree.
- I can identify and combine the like terms of a polynomial.

The word POLYNOMIAL can be broken down into two parts:
Poly, which is greek for the word “many”
Nomial, which is Greek for the word “names” but in math it is considered “terms”
Polynomial therefore means many terms.

A polynomial has many different parts to it all with different names.

Definitions:

*Algebraic expression* is a mathematical phrase which includes numbers and variables connected together by operations of $+$, $-$.  
Example:

*Algebraic terms* are the parts of an algebraic expression separated by operations $(+, -)$.  
Example:

*Variable* is a letter or symbol that represents a numeric quantity that can vary or change.  
Example:

*Coefficient* is the numerical factor of a variable term; it is the number in front of the variable.  
Example:

*Constant* is a symbol representing a value that does not change.  
Example:

*Note:* When you have a variable by itself, a coefficient of 1 is implied.  
For example, the term $m$, has a coefficient of 1.
### Class Examples:
For each of the following polynomials list the number of terms, variables, coefficients and constants.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Number of Terms</th>
<th>Variables</th>
<th>Coefficients</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4a - 4b - 24$</td>
<td>3</td>
<td>$a, b$</td>
<td>4, -4</td>
<td>24</td>
</tr>
<tr>
<td>$-14y + 24y - 3$</td>
<td>3</td>
<td>$y$</td>
<td>14, 24, -1</td>
<td>3</td>
</tr>
</tbody>
</table>

### Try on your own:
For each of the following polynomials list the number of terms, variables, coefficients and constants.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Number of Terms</th>
<th>Variables</th>
<th>Coefficients</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3a^2b + 8a + 7$</td>
<td>3</td>
<td>$a, b$</td>
<td>3, 8, 1</td>
<td>7</td>
</tr>
<tr>
<td>$4xy - y$</td>
<td>2</td>
<td>$x, y$</td>
<td>4, -1</td>
<td>0</td>
</tr>
</tbody>
</table>

1. How many terms are in each of the following expressions?
   a. $2x + 3y$
   b. $a + b - c$
   c. $3x^2$
   d. $(5x)(2y)$
   e. $3x^2 + 7x + 5$
   f. $-8x^3 + 5x^2 - 3x + 4$
   g. $(3x)(-2x^2)$
   h. $(6x^2) ÷ 2$

2. Circle the constant(s) in each of the expressions below.
   a. $3x + 4$
   b. $5x + 3y$
   c. $8x - 8$
   d. $5a - 3b + 2c + 9$
   e. $16x^2 - 5 + 3x$
   f. $7x^2 - 3x + 4$

3. Circle the variable(s) in each of the expressions below.
   a. $3a + 5b - 6$
   b. $5x + 15$
   c. $3x^2 - 4y^2 + 2z$
   d. $a + bc - 4c$
   e. $x^2y^2 - 3xy + 5$
   f. $-7y - 5z$

4. Circle the numerical coefficient(s) in each expressions below.
   a. $3xy$
   b. $3x + 4y$
   c. $-4abc$
   d. $2.6x^2 - 3.6y^2 + 7$
   e. $14xy^3$
   f. $x$

5. How many terms are in the expression $3 - 5xy + 3x^2y^3$ ? ____________________________

6. Write out the constant(s) in the expression $3x^2 - 5xy + 2$? ____________________________

7. Which is the numerical coefficient in the term $7x^2y^3z^4$ ? ____________________________
Introduction to Polynomials: Part II

Definitions:

Polynomial is a sum and/or difference of algebraic terms.

Examples:

Monomial is an algebraic expression with one term.

Examples:

Binomial is an algebraic expression with two terms.

Examples:

Trinomial is an algebraic expression with three terms.

Examples:

Class Examples:
Classify each of the following polynomials according to the number of terms.

| -24 − 4b − 4a | -24y − 18x^2y |
| -15 | 3y − x^2y − 11x^2 − 32 |
**Try on your own:**
Classify each of the following polynomials according to the number of terms.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2abc)</td>
<td>(x - 3)</td>
</tr>
<tr>
<td>(-2abc - 4b - 4a + 24)</td>
<td>(3c - b - 56)</td>
</tr>
</tbody>
</table>

**Ordering Polynomials**

**Definitions:**

- **Degree of a monomial** is the sum of the exponents of all the variables.

  Example:
  \(3x^3y^4\) has a degree

- **Degree of a polynomial in one variable** is the degree of the term with the highest exponent.

  Example:
  \(2x^4 + 3x^5\) has a degree of

- **Degree of a polynomial in two or more variables** is the highest sum of the exponents among the terms.

  Example:
  \(x^2y^2z^2 - 3x^4y + 4\), has a degree of

**Class Examples:**
State the degree of the following.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4a^5b^4 - 24 a^2b^3 + 4ab)</td>
<td>(-2y + 18x^2y)</td>
</tr>
</tbody>
</table>
Try on your own:
State the degree of the following.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5ab^3 c)</td>
<td>(6x^2y + 24y^4 - b^3 - a)</td>
</tr>
<tr>
<td>(-b^3)</td>
<td>(9x^3y + 24y^4 - b^6 - a)</td>
</tr>
</tbody>
</table>

Definitions:

**Descending order** is when the terms of a polynomial are arranged so that the powers of one variable are arranged from largest to smallest powers of the variable. This can also be referred to as STANDARD FORM as most polynomials are written in this form.

Example:

\(4x - y^3 + 5x^2y\) arranged in descending powers of \(x\) would be:

Ascending order is when the terms of a polynomial are arranged so that the powers of one variable are arranged from smallest to largest powers of the variable.

Example:

\(3x^2y - 3x^3 + 4x\) arranged in ascending powers of \(x\) would be:

Note: When the variables have the same power you then order them according the next variable in alphabetical order.

Class Examples:

Arrange the terms in descending powers of \(x\).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-xy^4 - 4 + 4x^3y)</td>
<td>(2x^2y + 18y^4 - x^3 - x)</td>
</tr>
</tbody>
</table>

Try on your own:

Arrange the terms in ascending powers of \(x\).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5xy^3z - xy^4 - 4 + 4x^3y)</td>
<td>(6x^2yz + 24xy^4 - x^3 - y)</td>
</tr>
</tbody>
</table>
Introduction to Polynomials: Part II

Assignment

1. How many terms are in each of the following expressions?
   a. \(3x^2 + 5x - 7\)  
   b. \(ab + bc\)  
   c. 7  
   d. \((x)(5)\)  
   e. \((3x^2)(2x)(3)\)  
   f. \(7x^3 - 3x^2 + 5x\)  
   g. \(2a + 3b - 4c\)  
   h. \((7x)(2y)\)  
   j. \((3x^2)^3\)  
   k. \(-7\)

2. State the degree of each of the following polynomials.
   a. \(3x^2\)  
   b. \(5a^8\)  
   c. \(30a^4 + 15a^3\)  
   d. \(6x^2 - 3x + 2\)  
   e. \(x^3 - y^2\)  
   f. \(-5x^3 + 3x\)  
   g. \(x^2y^3\)  
   h. \(5x\)  
   i. \(3xy\)  
   j. \(3x^5 + 4 - 5x^2y^2\)

3. Name the 2\(^{nd}\) degree term in the expression \(3x^3 + 5x^2 - 2x\). __________________________

4. Write any 4\(^{th}\) degree binomial __________________________

5. Write the following polynomials in standard form:
   a) \(7x^3 - 5x^6 + 2x^2 - 8x^5 + 23\) __________________________
   b) \(8x^5 + 7x^4 - 6x^5x^3 + 9x^9\) __________________________

6. Write out the 2\(^{nd}\) degree term of: \(6x^3 + 3x^3x^2 + 5xy - 7\) __________________________

7. Write the 3\(^{rd}\) degree term of: \(4x^3x^2 - 7x^3 + x^2\) __________________________

8. Write any 3\(^{rd}\) degree trinomial in standard form. __________________________

9. Using the expression \((-8x^2 + 3x - 5)\), answer the following questions.
   a) It contains _____________ terms, and is therefore called a ________________________
   b) The second term contains ___________ factors and they are _______________________
   c) This polynomials is written in the ________________________________ degree.
   d) The constant(s) in this expression is (are) ______________________________
   e) The numerical coefficient of the 2\(^{nd}\) term is ______________________________

10. Write any 5\(^{th}\) degree monomial. __________________________

11. Complete the polynomial by writing a 2\(^{nd}\) degree term. \(8x^7 + 4x^5 + \underline{\ } + 9x\)

12. Complete the polynomial by writing a 3\(^{rd}\) degree term. \(7x^6 + 5x^5 - \underline{\ } + 6x\)
Introduction to Polynomials Part III

Identifying and combining like terms:

We can add or subtract monomials only if the variable and the attached exponent are exactly the same. In each case we add or subtract the numerical coefficients and keep the variable and exponents the same.

Definition:
Like terms are the terms that have identical variables raised to identical exponents.

Examples:

Note: The sign before the coefficient goes with the term.

Class Examples:
Combine like terms for the following examples.

<table>
<thead>
<tr>
<th>$4x + 3x$</th>
<th>$12x + 6 - 4x + 13$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xy^3z - 12xy^3z + 3xy^3$</td>
<td>$2x + 5x^2 - 10x - 4x^2 + 8x$</td>
</tr>
</tbody>
</table>

Try on your own:

| $4x^2 + 7x + 3x^2 + 8 - 6x - 4$ | $5x + 3x^2 + 12x - 7x^2$ |
Introduction to Polynomials Part III

Assignment

Simplify the following expressions by adding or subtracting like terms.

1. $13x + 7x$
2. $14x^3 + 5x^5$
3. $-7x^2 - 5x^2 - 14x^2$

4. $-5x^2y + 2x^2y - 10x^2y$
5. $-6mn^2 + 15mn^2 - 12mn^2$
6. $17y - 2 + 8y - 9$

7. $5x + 8b + 19x + 5b - 9b$
8. $18n + 4 - 15n - 6$
9. $-5x + 17x - 8x$

10. $2xy^2 + \frac{1}{2}xy^2$
11. $7x^2 + x^2$
12. $-4y^3 + 6y^3 - 3y^3$

13. $xy^3z - 14xy^3z + 5xy^3$
14. $-8x^2 + 16x + 9x^2 - 16x$
15. $10y^3 - 16 + 12y - y^3$

16. $3x^2 + 5x^3 + 14x^2 - 6x^3$
17. $12x - 5x^2 + 24 + 8x$
18. $\frac{3}{4}x^2 - 2x^2 + \frac{1}{2}x + 4x^2$

19. $3x^3 - 7x^3 + 10x^2 + 13x^2$
20. $0.35x + 0.6x^3 + 0.7x + x^3$
21. $7x^4 + 0.3y^3 - 0.4y^4 + y^3$
Unit 4: Polynomials

Adding and Subtracting Polynomials

Concepts:
- Identify and combine like terms.
- Add & subtract polynomials and list in descending order of degree.

Self-Check:
- I can identify and combine like terms.
- I can add polynomial expressions.
- I can subtract polynomial expressions.
- I can list the terms of a polynomial in descending order of degree.

Combining like terms means to add terms where both the variables and the exponents are the same, by adding their coefficients together.

For example, since $3x^4y$ and $-5x^4y$ are like terms we could add them together to make $-2x^4y$.

Adding Polynomials:
- When combining like terms, add the coefficients.
- The variable part of the term stays the same.

Steps:
1. Remove the brackets.
2. Collect like terms and add.

Example: $(x^2 + 4x - 5) + (2x^2 - 5x + 1)$

Step One: Remove the brackets.
$x^2 + 4x - 5 + 2x^2 - 5x + 1$

Step Two: Collect like terms and add.
$x^2 + 2x^2 + 4x - 5x - 5 + 1$
$3x^2 - x - 4$

Class Examples:
Add the following.

| $(4x^2y + 4x - 5) + (2x^2y - 6x + 2)$ | $(6x^2 - 3) + (2x^2 - 5x + 8)$ |
Try on your own:
Add the following.

\[
\begin{array}{c}
(xy + 4x^3 - 5x) + (3xyz - xy^3 - 5x) \\
(xyz + x) + (-7xyz - 2x)
\end{array}
\]

Subtracting Polynomials:

**Steps:**
1. Keep the first polynomial the same, change the operation to addition and the second polynomial to its additive inverse.
2. Remove the brackets.
3. Collect like terms and add.

**Example:**
\[
(x^2 + 4x - 5) - (3x^2 + 4x + 4)
\]

**Step One:**
Keep the first polynomial the same, change the operation to addition and the second polynomial to its additive inverse.
\[
(x^2 + 4x - 5) + (-3x^2 - 4x - 4)
\]

**Step Two:**
Remove the brackets.
\[
x^2 + 4x - 5 + 3x^2 + 4x + 4
\]

**Step Three:**
Collect like terms and add.
\[
x^2 + 3x^2 + 4x + 4x - 5 + 4
\]
\[
4x^2 + 8x - 1
\]

**Class Examples:**
Subtract the following.

\[
\begin{array}{c}
(-4x^2y + 4x - 5) - (2x^3y - 6x + 2) \\
(-x^2 - 3) - (-5x^2 - 4x + 1)
\end{array}
\]
Try on your own:
Subtract the following.

\[(xyz + 4xy^3 - 5x) - (xyz - 4xy^3 - 5x)\]
\[(xyz + x) - (-7xyz - 2x + 5)\]

---

**Subtracting Like Terms**

We can also Subtract Like Terms

Suppose that we have bought 5 apples and 6 bananas, but we eat two bananas before putting our fruit into the bowl.

\[\text{The Algebra is: } 5a + 6b - 2b\]
\[= 5a + 6b - 2b \quad (6 \text{ bananas take away } 2 \text{ is } 4)\]
\[= 5a + 4b\]
\[= 5a + 4b \checkmark\]

---

CLASS ACTIVITY

CARD GAME:
Polynomials UNO: PUNO

ROW GAME
Adding and Subtracting Polynomials

A: Add the following polynomials

1. $(3x^2 + 5x) + (7x^2 + 2x)$
2. $(5x - 3) + (7x + 4)$

3. $(8x - 3) + (-3x - 8)$
4. $(-6x + 2) + (3x - 5)$

5. $(8x - 2) + (3x + 5)$
6. $(-9x^2 + 2x) + (8x^2 + 2)$

7. $(-8x + 2) + (7x - 4)$
8. $(2x + 8) + (7x - 8)$

9. $(2x^2 + 3x + 5) + (6x^2 + 4x + 5)$
10. $(3x^2 - 8x - 6) + (7x^2 + 8x - 4)$

11. $(9x^2 - 8x - 4) + (-9x^2 + 8x + 6)$
12. $(7x - 4) + (3x + 2) + (8x - 5)$

13. $(-8x - 3) + (7x - 2) + (-3x - 7)$
14. $(7x + 2) + (3x^2 - 4x) + (2x - 1)$

15. $(3x^2 + 2) + (7x - 8) + (4x^2 + 5)$
16. $(3x - 5) + (-3x - 5) + (8x - 3)$
B: Subtract the following Polynomials

1. \((5x + 8) - (4x + 3)\)  
2. \((7x - 4) - (3x + 5)\)

3. \((8x - 9) - (3x - 4)\)  
4. \((6x - 8) - (-8x - 3)\)

5. \((3x^2 - 5x) - (-6x^2 + 2)\)  
6. \((2 - 7x) - (3 - 8x)\)

7. \((7x^2 - 5x + 3) - (3x^2 - 8x - 4)\)  
8. \((3x - 2) - (5x - 3) - (8x - 2)\)

9. \((3x - 4) - (8x - 2) - (3x) + 2\)  
10. \((9x^2 - 5x + 7) - (3x^2 + 8x)\)

11. \((7x^2 - 5x + 2) - (3x + 8)\)  
12. \((-9x^2 - 3x + 4) - (-8x^2 + 7x)\)

13. \((3 - 2x + 5x^2) - (-8 - 7x + 2x^2)\)  
14. \((3a - 5b - 6c) - (2a - 5b - 6c)\)

C: Solve the following perimeter problems

1. Find the perimeter of a triangle whose sides are \((3x + 6), (2x - 5)\) and \((8x - 5)\) metres.

2. Find the perimeter of a square whose sides are \((3x + 5)\) meters.

3. Find the perimeter of a rectangle whose length is \((7x - 8) m\) and width is \((2x + 3) m\).
D: Add or subtract the following polynomials

1. \((-x + 12a) + (7x - 8a) + (5a - 2) + (6x + 7)\)
2. \((4x^2 + 3x) + (3x^2 - 2x) + (2x + 3b) + (4x - 7b)\)

3. \((2y^3 - 3y) + (y^3 + 3y) + (y^3 - 10y) - (6y + 7) + (y + 4)\)
4. \((7y - 6) + (5y - 2) - (-6y - 1)\)

5. \((y - 5) - (6y + 4) + (7y - 6)\)
6. \((x - 2) + (5x - 7) - (2x - 3)\)

7. \((5x + 7a) + (5x + 2a) - (7x - 3a)\)
8. \((2x + 3) - (-3y - 2) - (3y + 1)\)

9. \((5x^3 - 2x - 3) + (6x^3 + x^2 + x) + (4x^2 + 3)\)
10. \((5a - 6c - 7d) + (8a - 9c - 10d) + (11a - 12c - 13d)\)
Unit 4: Polynomials
Lesson 19

Multiplication: Exponents Laws, Monomials and Binomials

Concepts:
- Review exponent laws and use in polynomial expressions.
- Model, record and perform the operation of multiplication on monomials and binomials.

Self-Check:
- I can apply the exponent laws to polynomial expressions.
- I can multiply a monomial by a monomial.
- I can multiply a monomial by a binomial.
- I can multiply a binomial by a binomial.

Review from Unit 1:

Multiplying powers when the bases are the same you add exponents.

\[ a^n \times a^m = a^{n+m} \]

\[ 3^4 \times 3^5 = 3^9 \]

This also could look like \( \left(3^4\right)\left(3^5\right) = 3^9 \) or \( 3^4 \cdot 3^5 = 3^9 \)

\{both mean multiplication\}

Dividing powers when the bases are the same you subtract exponents

\[ a^n \div a^m = a^{n-m} \]

\[ 3^7 \div 3^4 = 3^3 \]

This could also look like \( \frac{3^7}{3^4} = 3^3 \)

* The same rules will apply when multiplying or dividing monomials, binomials, and trinomials.

Multiplying Monomials by Monomials

- Multiply coefficients together and variables together.
- Add exponents when multiplying numbers with the same base.

**Example:** Multiply \((4x)(-3x)\)

\[-12x^2 \quad \text{Because} \quad 4 \times -3 = -12 \quad \text{and} \quad x^{1+1} = x^2\]

**Example:** Multiply \((4x^2y^3)(5xy^2)\)

\[20x^3y^5 \quad \text{Because} \quad 5 \times 5 = 20 \quad \text{and} \quad x^{2+1} = x^3, \quad y^{3+2} = y^5\]
Class Examples:
Multiply the following.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-7x^3y)(2x^6y))</td>
<td>((-x^2y)(x^6y^4))</td>
</tr>
</tbody>
</table>

Try on your own:
Multiply the following.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-4acb^3)(-2ax^2bc^4))</td>
<td>((9x^3y^2z)(9x^3y^2z))</td>
</tr>
</tbody>
</table>
Multiplying Polynomials: Part I

A: Find the product of the following monomials.

1. \((6a)(5a)\) 2. \((7x)(3x)\) 3. \((xy)(yz)\)

4. \((3x^2)(5x^2)\) 5. \((6x^3)(3x^2)\) 6. \((8x)(-4)\)

7. \((3x)(5y)(7z)\) 8. \((2a)(5ab)(4bc)\) 9. \(6(3x^2y)\)

10. \(ab \cdot 5ab \cdot cd\) 11. \(x \cdot xy \cdot xyz\) 12. \(x^3 \cdot x^2y \cdot xyz\)

13. \(8x^3y \cdot -5x^2\) 14. \(-3x \cdot -4xy \cdot -2xyz\) 15. \(-3.4a^2 \times 5b^2\)

16. \(\left(\frac{3}{4} x^2y\right) \left(\frac{1}{2} xy^2\right)\) 17. \((-15xy) \left(\frac{3}{5} xy^2\right) \left(\frac{2}{3} x\right)\)

18. \((27x^2y)(-3x) \left(\frac{2}{3} x\right)\) 19. \(\left(\frac{1}{2} b^2 c\right) \left(\frac{4}{3} b^2 c^2\right) \left(\frac{-4}{5} b^3 c^2\right)\)

Assignment
Multiplying a Polynomial by a Monomial

Method #1: Using Algebra Tiles

Steps:
1. Draw a frame with a square corner.
2. Place one of the monomials along the horizontal frame line. Place the other monomial along the vertical frame line.
3. Fill the outlined rectangular space.
4. Write the algebraic expression that represents the area of the rectangular space.

Example: $3(x + 2)$

Step One: Draw a frame with a square corner.

Step Two: Place one of the monomials along the horizontal frame line. Place the other monomial along the vertical frame line.

Step Three: Fill the outlined rectangular space.

Step Four: Write the algebraic expression that represents the area of the rectangular space.

$3x + 6$

REMINDER:

These shapes represent the following values:
Example: \((2x)(-x - 1)\)

Step One: Draw a frame with a square corner.

Step Two: Place one of the monomials along the horizontal frame line. Place the other monomial along the vertical frame line.

Step Three: Fill the outlined rectangular space.

Step Four: Write the algebraic expression that represents the area of the rectangular space.

\(-2x^2 - 2x\)

Class Example:
Multiply with tiles.

\((4)(3x - 2)\)
Try on your own:
Multiply with tiles.

\((-2x)(x - 3)\)

Method #2: Distributive property:

Distributive Property allows you to remove the brackets by multiplying each of the terms inside the brackets by the term outside of the brackets.

For example, in the expression \(2(x + 4)\) you multiply both the variable term and the constant by 2.

\[
2(x + 4) = (2)(x) + (2)(4) \\
= 2x + 8
\]

Remember: To multiply variables with the same base, you keep the base and add the exponents.

Steps:

1. Use the distributive property to expand each expression. Multiply each term inside of the brackets by the term on the outside.

2. Collect like terms (simplify).

Example: \(3x(x^2 + x - 4)\)

Step One: Use the distributive property to expand each expression.

\[
3x(x^2 + x - 4) = 3x^3 + 3x^2 - 12x
\]

Step Two: Collect like terms (simplify).

It is already simplified because there are no like terms.
**Example:** \[3x(x - 3) - 4(x + 3)\]

**Step One:** Use the distributive property to expand each expression.

\[
3x(x - 3) - 4(x + 3) \\
\Rightarrow 3x^2 - 9x - 4x - 12
\]

**Step Two:** Collect like terms (simplify).

\[3x^2 - 13x - 12\]

**Class Examples:**
Expand and simplify the following.

<table>
<thead>
<tr>
<th>[m(3m^2 - 10m - 14)]</th>
<th>[x(x - 6) - 3x(x + 2)]</th>
</tr>
</thead>
</table>

**Try on your own:**
Expand and simplify the following.

<table>
<thead>
<tr>
<th>[-4a(6a^2 - 6a - 6)]</th>
<th>[5y(3 - y) - 5y(3 + y)]</th>
</tr>
</thead>
</table>
Multiplying Polynomials: Part II

A: Multiply & Simplify the following expressions:

1. \(6(3x + 4)\)  
2. \(2x(4x - 1)\)

3. \(-3x(2x^2 + 4x - 5)\)  
4. \(5xy(x^2 + 3xy + 2y^2)\)

5. \(2a^2(3a^2 - 14a - 9)\)  
6. \(8ab^3(ab^2 + a^4b - a^5)\)

7. \(-5x^2y^2(2x^2y - xy + 7xy^2)\)  
8. \(8x(2x^4 + x^3 - 5x^2 - 1)\)
Multiplying Binomials by Binomials

Method #1: Using Algebra Tiles

Steps: 1. Draw a frame with a square corner.
2. Place one of the binomials along the horizontal frame line. Place the other binomial along the vertical frame line.
3. Fill the outlined rectangular space.
4. Write the algebraic expression that represents the area of the rectangular space.

Example: \((x + 2)(x + 3)\)

Step One: Draw a frame with a square corner.

Step Two: Place one of the monomials along the vertical frame line. Place the other monomial along the horizontal frame line.

Step Three: Fill the outlined rectangular space.

Step Four: Write the algebraic expression that represents the area of the rectangular space.

\[x^2 + 5x + 6\]

Note: \((+) \times (+) = (+)\)
Class Example:
Use tiles to find the product.

\((x + 1)(x - 3)\)

Try on your own:
Use tiles to find the product.

\((x - 2)(x - 2)\)

Method #2: Binomial Products

- You expand by using the distributive property.
- You multiply each of the terms in the first binomial by each of the terms in the second binomial.
- Collect like terms & simplify.

You can remember this method by using the acronym **FOIL**.

- **F**: First term × First term
- **O**: Outside term × Outside term
- **I**: Inside term × Inside term
- **L**: Last terms × Last term

\((x + 9)(x + 1)\)

<table>
<thead>
<tr>
<th>First: (x(x) = x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside: (x(1) = x)</td>
</tr>
<tr>
<td>Inside: (9(x) = 9x)</td>
</tr>
<tr>
<td>Last: (9(1) = 9)</td>
</tr>
</tbody>
</table>

\(x^2 + 1x + 9x + 9\)

\(x^2 + 10x + 9\)
**Example:**  
\[(x + 2)(x + 2)\]

F.O.I.L  
First terms \((x)(x) = x^2\)  
Outside terms \((2)(x) = 2x\)  
Inside terms \((2)(x) = 2x\)  
Last terms \((2)(2) = 4\)

\[x^2 + 2x + 2x + 4\]

\[x^2 + 4x + 4\]  
Collect like terms.

**Example:**  
\[2(3x + 2)(x - 4)\]

Use the distributive property to multiply the monomial and the binomial.

**Note:** 2 was only distributed to \(3x + 2\); the first set of brackets.

F.O.I.L  
First terms \((6x)(x) = 6x^2\)  
Outside terms \((6x)(-4) = -24x\)  
Inside terms \((4)(x) = 4x\)  
Last terms \((4)(-4) = -16\)

\[6x^2 - 24x + 4x - 16\]

\[x^2 - 20x - 16\]  
Collect like terms.

**Class Examples:**  
Multiply the following.

\[(5c - 4)^2\]  
\[3(2y - 2)(2y - 2)\]

**Try on your own:**  
Multiply the following.

\[(4a + 3)^2\]  
\[6(x - 4)(5x + 2)\]
Multiplying Polynomials: Part III

Assignment

A: Multiplying two Binomials

1. \((x + 7)(x + 4)\)  
2. \((x - 5)(x + 1)\)

3. \((x - 12)(x - 2)\)  
4. \((2x + 7)(3x + 4)\)

5. \((3x + 2)(4x - 1)\)  
6. \((7x - 3)(6x - 5)\)

7. \((x - 9)(2x + 7)\)  
8. \((x - 11)(5x - 2)\)

CLASS ACTIVITY

TARSIA Puzzle
Unit 4: Polynomials

Lesson 19

Dividing and Greatest Common Factoring

Concepts:
- Model, record and perform the operation of division on monomials.
- Identify and divide out common factors in polynomial expressions.

Self-Check:
- I can divide a monomial by a monomial.
- I can identify the greatest common factor in a polynomial expression.
- I can divide out the greatest common factor from a polynomial expression.

Dividing Monomials by Monomials

- Divide coefficients together and variables together.
- Subtract exponents when dividing numbers with the same base.

Example:
\[
\frac{-24x^2y^6}{3x^2y^4} = -8y^2
\]

Because
\[
24 \div 3 = -8
\]
\[
x^{2-2} = x^0
\]
\[
y^{6-4} = y^2
\]

Note: \( x^0 = 1 \) so we don’t need to include it.

Example:
\[
\frac{48x^2y^8}{36x^5y^4} = \frac{4y^4}{3x^3}
\]

Because
\[
48 \div 12 = \frac{4}{3}
\]
\[
36 \div 12 = \frac{3}{3}
\]
\[
x^{2-5} = x^{-3}
\]
\[
y^{8-4} = y^4
\]

Note: \( x^{-3} = \frac{1}{x^3} \)
### Class Examples:
Divide the following.

<table>
<thead>
<tr>
<th>((-84acb^3) ÷ (-12a^2bc^4))</th>
<th>((3x^{12}y^{15}) ÷ (9x^3y^3z^3))</th>
</tr>
</thead>
</table>

### Try on your own:
Divide the following.

| \((10a^6cb^3) ÷ (-2a^2bc^4)\) | \((-18acb^3) ÷ (-12a^2bc^4)\) |
## Dividing Monomials

A: Find the Quotient of each of the following.

1. \( \frac{8x}{2} \)

2. \( \frac{15a^2b^2}{5ab} \)

3. \( \frac{7x^2y}{7x^2y} \)

4. \( \frac{-36xy^4}{-6xy} \)

5. \( \frac{72x^3y^2}{9x^2y^4} \)

6. \( \frac{-10ab}{25a^2b^2c^2} \)

7. \( \frac{15x^2y}{45x^3y^3} \)

8. \( \frac{m^4p^5q^7}{m^3p^3q^6} \)

9. \( \frac{56a^2b^4c^3}{28ab^3c^2} \)

10. \( (-2x^2y^3) \div (-x^3y^4) \)

11. \( (18ab) \div (-6ab^2) \)

12. \( (15m^4) \div (30m^4) \)

13. \( (8a^4b^6c^2) \div (4a^4b^6c^2) \)

14. \( \left( \frac{3}{5} x^2y^3 \right) \div (10y^4) \)

15. \( \left( \frac{-3}{8} m^4p^3 \right) \div \left( \frac{8}{3} m^3p \right) \)
Factoring Expressions with Common Factors

This process is the opposite operation of the distributive property. We will be UNDOING the distributive property.

Steps:
1. Write each expression as a product of its prime factors and identify factors common to each expression.
2. Multiply the common factors together to calculate the GCF.
3. Multiply the GCF by the remaining factors in each monomial.
4. Check your answer by applying the distributive property.

Example: Factor the expression $6m^2n + 9mn$.

Step One: Write each expression as a product of its prime factors and identify factors common to each expression.

$$6m^2n = (2)(3)(m)(m)n$$
$$9mn = (3)(3)(m)n$$

Step Two: Multiply the common factors together to calculate the GCF.

$$3 \times m \times n = 3mn \quad GCF = 3mn$$

Step Three: Place the GCF outside of the brackets & divide the GCF by the remaining factors in each monomial.

$$3mn(2m + 3)$$

Step Four: Check your answer by applying the distributive property.

$$3mn(2m + 3) = 3mn(2m + 3) = 6m^2n + 9mn$$
Example: Factor the expression $2x^3 + 8x^2 - 4x$.

Step One: Write each expression as a product of its prime factors and identify factors common to each expression.

\[
\begin{align*}
2x^3 &= 2 \cdot 3 \cdot x \cdot x \cdot x \\
8x^2 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \\
4x &= 2 \cdot (-2) \cdot x
\end{align*}
\]

Step Two: Multiply the common factors together to calculate the GCF.

GCF = $2x$

Step Three: Multiply the GCF by the remaining factors in each monomial.

$2x(x^2 + 4x - 2)$

Step Four: Check your answer by applying the distributive property.

\[
2x^3 + 8x^2 - 4x = 2x(x^2 + 4x - 2) = 2x^3 + 8x^2 - 4x
\]

Class Examples:
Factor fully. Show a check.

\[
\begin{align*}
15a^3 - 10a^2 + 25a &= 5(3a^3 - 2a^2 + 5a) \\
-12y^3 - 3y^2 + 9y &= -3(4y^3 + y^2 - 3y)
\end{align*}
\]

Try on your own:
Factor fully. Show a check.

\[
\begin{align*}
20m^2 + 60mn + 45m &= 5(4m^2 + 12mn + 9m) \\
-3ac + 5bc - c^2 &= -(3a - 5b + c^2)
\end{align*}
\]
Factor each expression fully:

1. \( x^2y - xy^3 \)
2. \( 6x^2y^5 - 8x^3y^4 + 12x^4y^3 \)

3. \( 15g^3h^3 + 9g^4h^2 - 27g^2h \)
4. \( a^5b^4c^2 + a^3b^2c - a^2b \)

5. \( 4c^2 + 12c - 24 \)
6. \( 5y^3 - 20y^2 \)

7. \( 9x^3 + 15x^2 - 12x \)
8. \( 2n^2 + 8n - 48 \)

9. \( 14x^3y^2z - 28x^4y^3z^2 + 56x^5y^4z^3 \)
10. \( 10x^2 + 18xy + 8y^2 \)

Interactive Game:
Bump, Set, Spike!
This review will cover:
Lesson 17: Introduction to Polynomials
Lesson 18: Adding & Subtracting Polynomials
Lesson 19: Multiplying Polynomials
Lesson 20: Dividing Polynomials & Factoring Polynomials

1. Perform the operation and simplify the polynomial:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$(3x + y) + (−8y − 4)$</td>
</tr>
<tr>
<td>h.</td>
<td>$−4y − 5) + (−3y + 6)$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td>$3(2x^2 − 4x + 5) + (3x − 1)$</td>
</tr>
<tr>
<td>d.</td>
<td>$5b(2b + 5)$</td>
</tr>
<tr>
<td>e.</td>
<td>$(4x)(5xy)(−2y)$</td>
</tr>
<tr>
<td>f.</td>
<td>$(x − 9)(x − 2)$</td>
</tr>
<tr>
<td>g.</td>
<td>$(3a^2b^3) ÷ 3ab$</td>
</tr>
</tbody>
</table>
2. Determine the Perimeter & Area of the two following shapes:

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Rectangle" /></td>
<td><img src="image2" alt="Triangle" /></td>
</tr>
</tbody>
</table>

- **Perimeter:**
  - a) \(P_a = 2x + 4 + 2x + 2 = 4x + 6\)
  - b) \(P_b = x + 2 + 3x + 2 = 4x + 4\)

- **Area:**
  - a) \(A_a = (x + 4)(x + 2) = x^2 + 6x + 8\)
  - b) \(A_b = \frac{1}{2} \cdot 5x^2 + 2x \cdot 2 = \frac{1}{2} \cdot 11x^2 + 4x\)

3. Determine the volume of the following:

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Cube" /></td>
<td><img src="image4" alt="Cuboid" /></td>
</tr>
</tbody>
</table>

- **Volume:**
  - a) \(V_a = (x + 4)(x + 5) = x^2 + 9x + 20\)
  - b) \(V_b = 2x \cdot x \cdot (3x - 1) = 6x^3 - 2x^2\)

4. Andree’s backyard is a rectangle, and the Area is \(18xy\).
   He has measured the length of the lawn to be \(9x\). What is the width of the lawn?

   \[\text{Width} = \frac{\text{Area}}{\text{Length}} = \frac{18xy}{9x} = 2y\]
Unit 3: Polynomials

Factors Trinomials

Review of Integers

Adding/Subtracting Integers

1. Adding two positive integers

   Example: \((+3) + (+2) = (+5)\)

   - The sign will always be + because there are only positive tiles.
   - The numeric value will be the sum of the tiles.

2. Adding two negative integers

   Example: \((-7) + (-5) = (-12)\)

   - The sign will always be – because there are only negative tiles.
   - The numeric value will be the sum of the tiles.

3. Adding a positive and a negative integer.

   Example: \((-3) + (+4) = (+1)\)

   - The sign will be the sign of the larger number.
   - The numeric value will be the difference between the number of shaded and un-shaded tiles.

Class Examples:
Add the following.

<table>
<thead>
<tr>
<th>(-10) + (+7)</th>
<th>(+11) + (+2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2) + (-20)</td>
<td>(+5) + (-5)</td>
</tr>
</tbody>
</table>
**Try on your own:**
Add the following.

| (+9) + (-4) | (-11) + (-11) |
| (-11) + (+11) | (0) + (+30) |

**Multiplying Integers**

**Steps:**

1. Find the sign part of the answer.
   
   (+) × (+) = (+)  
   (+) × (−) = (−)  
   (−) × (+) = (−)  
   (−) × (−) = (+)  

2. Find the number part of the answer by multiplying the 2 numbers together.

3. State the final answer (sign and numeric value).

**Example:**  
(+9) × (+3)  
(+ × (+) = (+)  
9 × 3 = 27  
(+9) × (+3) = + 27  

**Class Examples:**
Multiply the following.

| (-4) × (+2) | (+3) × (-7) |
| (+3) × (+6) | (-7) × (-7) |

**Try on your own:**
Multiply the following.

| (+12) × (+10) | (-11) × (+5) |
| (-11) × (-3) | (+22) × (-3) |
Finding Factors of Trinomials:

Steps:
1. List all of the factors of the number (product).
2. Identify the two factors that give you the indicated sum and product.
3. State the final answer.

Example: Find two integers whose sum is +7 and product is +12.

Step 1: List all of the factors of the number (product).

\[12 = 1, 2, 3, 4, 6, 12\]

Step 2: Identify the two factors that give you the indicated sum and product.

\[\underline{\text{____} + \underline{\text{____}} = 7}\]
\[\underline{\text{____} \times \underline{\text{____}} = 12}\]

\[3 + 4 = +7\]
\[3 \times 4 = +12\]

Step 3: State the final answer.

+3, +4 are the two integers which sum to 7 and multiply to 12.

Class Examples:
Find the integers that satisfy the following conditions.

<table>
<thead>
<tr>
<th>sum is +6</th>
<th>sum is -6</th>
</tr>
</thead>
<tbody>
<tr>
<td>product is +8</td>
<td>product is -16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sum is -4</th>
<th>sum is +6</th>
</tr>
</thead>
<tbody>
<tr>
<td>product is +3</td>
<td>product is +5</td>
</tr>
</tbody>
</table>
**Try on your own:**
Find the integers that satisfy the following conditions.

<table>
<thead>
<tr>
<th>sum is +8</th>
<th>sum is +10</th>
</tr>
</thead>
<tbody>
<tr>
<td>product is +12</td>
<td>product is -11</td>
</tr>
<tr>
<td>sum is -12</td>
<td>sum is -5</td>
</tr>
<tr>
<td>product is +35</td>
<td>product is -24</td>
</tr>
</tbody>
</table>
Assignment

1. Find the missing integer.
   
a. \[ \text{___________} + (+4) = (-13) \]
   b. \[ (-7) + \text{___________} = (-11) \]
   
c. \[ \text{___________} + (-11) = (-6) \]
   d. \[ (+10) + \text{___________} = (+3) \]
   
e. \[ \text{___________} + (-26) = (-10) \]
   f. \[ (-16) + \text{___________} = (-27) \]
   
g. \[ (+6) \times \text{___________} = +42 \]
   h. \[ (+11) \times \text{___________} = (-99) \]

   i. \[ \text{___________} \times (-8) = (-32) \]
   j. \[ \text{___________} \times (-4) = (+36) \]

2. Find the integers that satisfy the following conditions.

<table>
<thead>
<tr>
<th>sum is -4</th>
<th>product is +4</th>
<th>sum is -9</th>
<th>product is +18</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>4</td>
<td>-7</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sum is -10</th>
<th>product is +25</th>
<th>sum is -46</th>
<th>product is +45</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15</td>
<td>25</td>
<td>-10</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sum is +5</th>
<th>product is -24</th>
<th>sum is -2</th>
<th>product is -24</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-24</td>
<td>-3</td>
<td>-24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sum is +17</th>
<th>product is +30</th>
<th>sum is -5</th>
<th>product is -14</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>30</td>
<td>-7</td>
<td>-14</td>
</tr>
</tbody>
</table>
3. Find the integers that satisfy the following conditions.

<table>
<thead>
<tr>
<th>Sum is</th>
<th>Product is</th>
<th>Sum is</th>
<th>Product is</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-6</td>
<td>+1</td>
<td>-6</td>
</tr>
<tr>
<td>-5</td>
<td>-50</td>
<td>-5</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-42</td>
<td>-4</td>
<td>-5</td>
</tr>
<tr>
<td>+2</td>
<td>-24</td>
<td>+13</td>
<td>+40</td>
</tr>
<tr>
<td>+12</td>
<td>+11</td>
<td>+16</td>
<td>+15</td>
</tr>
<tr>
<td>+11</td>
<td>+24</td>
<td>-1</td>
<td>-20</td>
</tr>
</tbody>
</table>
Factoring Trinomials of the form: $x^2 + bx + c$

**Steps:**
1. Find two numbers that multiply to “$c$” and add to “$b$”.
2. Check.
3. Draw two pairs of brackets. In the first brackets place $x$ and your first number (if the number is $+$ then write $x +$ the number, if the number is $-$ then write $x -$ the number).
4. In the second brackets place $x$ and your second number (if the number is $+$ then write $x +$ the number, if the number is $-$ then write $x -$ the number).
5. Check by using FOIL.

**Example:** Factor $x^2 + 7x + 10$.

**Step One:** Find two numbers that multiply to “$c$” and add to “$b$”.

+$5$ and $+2$

**Step Two:** Check.

$5 \times 2 = 10 \quad 5 + 2 = 7$

**Step Three:** Draw two pairs of brackets.
In the first brackets place $+5$.

$(x + 5)$ ( 

**Step Four:** In the second brackets place $+2$.

$(x + 5) (x + 2)$

**Step Five:** Check by using FOIL.

FOIL

$(x + 5) (x + 2)$

$x^2 + 5x + 2x + 10$

$x^2 + 7x + 10$
Example: Factor \( x^2 + 4x - 21 \).

Step One: Find two numbers that multiply to “c” and add to “b”.

-3 and +7

Step Two: Check.

\((-3) \times (+7) = -21\sqrt{\ }
\
\((-3) + (+7) = +4\sqrt{\ }

Step Three: Draw two pairs of brackets.

In the first brackets place \( x \) and -3

\(( x - 3 ) ( \quad )\)

Step Four: In the second brackets place \( x \) and +7

\(( x - 3 ) ( x + 7 )\)

Step Five: Check by using FOIL.

\( (x - 3) (x + 7) \)

\( x^2 + 7x - 3x - 21 \)

\( x^2 + 4x - 21 \)

Note: It does not matter which factor goes first or second.

Class Examples:
Factor fully. Show a check.

\( x^2 + 3x - 40 \)

\( x^2 - x - 30 \)

Try on your own:
Factor fully. Show a check.

\( w^2 + 5w - 24 \)

\( y^2 - 12y + 27 \)
Factoring Trinomials

A: Factor each of the following fully.

1. $2xy - 8xy^2$

2. $6x^2 - 30x + 24$

3. $15g^3h^3 + 9g^4h^2 - 27g^2h$

4. $x^2 - 15x + 54$

5. $4c^2 + 12c - 72$

6. $5y^2 - 20y$

7. $t^2 + 7t - 44$

8. $n^2 + 8n - 48$

9. $2w^2 - 2w - 60$

10. $x^2 + 9xy + 8y^2$

11. $x^2 - 8x - 20$

12. $x^2 - 8x + 12$

13. $p^2 - 16p - 36$

14. $m^2 + 15m + 44$
B: Find all possible integers to replace $A$ so that each trinomial can be factored.

1. $x^2 + Ax + 10$
2. $b^2 + Ab - 9$
3. $y^2 + Ay - 12$

C: Find one possible integer to replace $k$ so that each trinomial can be factored.

1. $x^2 + 7x + k$
2. $y^2 - 4y + k$